FINDING THE EQUATION OF A CIRCLE

Definitions:

- **Circle**: is the set of all points in a plane that lie a fixed distance from a fixed point. The fixed distance is called the **radius** and the fixed point is called the **center** of the circle.

Important Properties:

- **Equation of a circle**: An equation of the circle with center (h, k) and radius r is given by

$$\boxed{(x-h)^2 + (y-k)^2 = r^2.}$$

  This is called the standard form for a circle.

- Note to find the equation of a circle you need two items: the center and the radius.

Common Mistakes to Avoid:

- In the equation of a circle, make sure that you subtract the $x-$coordinate and the $y-$coordinate of the center.
- In the equation of a circle, do not forget to square the radius.

PROBLEMS

1. **Given the following circle equations, determine the center and radius**.

(a) $(x-3)^2 + (y-5)^2 = 16$

$$r^2 = 16$$

$$\sqrt{r^2} = \sqrt{16}$$

$$r = 4$$

$$\boxed{\text{Center} = (3, 5), \qquad r = 4}$$

(b) $(x+6)^2 + y^2 = 10$

$$r^2 = 10$$

$$\sqrt{r^2} = \sqrt{10}$$

$$r = \sqrt{10}$$

$$\boxed{\text{Center} = (-6, 0), \qquad r = \sqrt{10}}$$

2. **Find the equation of the circle with center $(-2, 3)$ and radius $r = 2$.**

Here we know that $(h, k) = (-2, 3)$ and $r = 2$. Therefore, substituting this information into the equation of a circle, we get

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-2))^2 + (y - 3)^2 = 2^2$$
$$(x + 2)^2 + (y - 3)^2 = 4$$
$$\boxed{(x + 2)^2 + (y - 3)^2 = 4}$$

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3. **Find the equation of the circle with center $(5, -2)$ and radius $r = \sqrt{7}$.**

Here we know that $(h, k) = (5, -2)$ and $r = \sqrt{7}$. Therefore, substituting this information into the equation of a circle, we get

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 5)^2 + (y - (-2))^2 = \left(\sqrt{7}\right)^2$$
$$(x - 5)^2 + (y + 2)^2 = 7$$
$$\boxed{(x - 5)^2 + (y + 2)^2 = 7}$$

4. **Find the equation of the circle with center $(-7, 3)$ and passes through $(4, -1)$.**

Here we know that $(h, k) = (-7, 3)$ but we are not given the radius. However, we can find the radius by using the distance formula. Therefore,

$$\begin{aligned} r &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(-7 - 4)^2 + (3 - (-1))^2} \\ &= \sqrt{(-11)^2 + (4)^2} \\ &= \sqrt{121 + 16} \\ &= \sqrt{137} \end{aligned}$$

Now substituting all of this information into our equation of a circle we get

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-7))^2 + (y - 3)^2 = \left(\sqrt{137}\right)^2$$
$$(x + 7)^2 + (y - 3)^2 = 137$$
$$\boxed{(x + 7)^2 + (y - 3)^2 = 137}$$

5. **Find the equation of the circle which passes through the origin and $(4, -8)$.**

Remember that the origin is $(0, 0)$. On this problem we need to find the center and the radius.

To find the center C, we will use the midpoint formula since the center must lie equidistant from the two given points.

$$\begin{aligned} C &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{0 + 4}{2}, \frac{0 + -8}{2}\right) \\ &= \left(\frac{4}{2}, \frac{-8}{2}\right) \\ &= (2, -4) \end{aligned}$$

Now that we have the center, we can find the radius by using the distance formula on the center $(2, -4)$ and either the point $(0, 0)$ or $(4, -8)$. We will use $(0, 0)$.

$$\begin{aligned} r &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(2 - 0)^2 + (-4 - 0)^2} \\ &= \sqrt{2^2 + (-4)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20}. \end{aligned}$$

Finally, substituting $(h, k) = (2, -4)$ and $r = \sqrt{20}$ into the equation for a circle, we get

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ (x - 2)^2 + (y - (-4))^2 &= \left(\sqrt{20}\right)^2 \\ (x - 2)^2 + (y + 4)^2 &= 20 \end{aligned}$$

$$\boxed{(x - 2)^2 + (y + 4)^2 = 20}$$

6. **Find the equation of the circle with endpoints of the diameter at $(-1, 1)$ and $(7, 9)$.**

Once again, we need to find both the center and the radius. To find the center C, we will use the midpoint formula since the center must lie equidistant from the two given points.

$$\begin{aligned} C &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{-1 + 7}{2}, \frac{1 + 9}{2}\right) \\ &= \left(\frac{6}{2}, \frac{10}{2}\right) \\ &= (3, 5) \end{aligned}$$

Next, to find the radius we will use the distance formula on the center $(h, k) = (3, 5)$ and either one of the given points. We will use $(7, 9)$.

$$\begin{aligned} r &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(3 - 7)^2 + (5 - 9)^2} \\ &= \sqrt{(-4)^2 + (-4)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32}. \end{aligned}$$

Finally, since we know that $(h, k) = (3, 5)$ and $r = \sqrt{32}$, substituting into the equation of a circle we find that

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ (x - 3)^2 + (y - 5)^2 &= \left(\sqrt{32}\right)^2 \\ (x - 3)^2 + (y - 5)^2 &= 32 \end{aligned}$$

$$\boxed{(x - 3)^2 + (y - 5)^2 = 32}$$