Definition:

- **Exponential function**: For $a > 0$, the exponential function with base a is defined by

$$\boxed{f(x) = a^x}$$

Guidelines for solving exponential equations:

1. Isolate the exponential expression on one side of the equation. (If the exponential expression cannot be isolated, try factoring the equation. Then set each factor equal to zero and continue to step 2.)
2. Take the logarithm of each side in order to "bring down" the variable in the exponent. (It does not matter which base you use on the logarithm as long as it is the same on both sides.)
3. Solve for the variable.

Common Mistakes to Avoid:

- Do NOT take the logarithm of both sides of the expression until the exponential is isolated. For example, $\ln 3e^{2x} \neq 6x \ln e$. Instead, $\ln 3e^{2x} = \ln 3 + \ln e^{2x} = \ln 3 + 2x \ln 3$.
- When "bringing down" an exponent be careful to distribute. For example,

$$\log 2^{3x+1} = (3x + 1) \log 2 = 3x \log 2 + \log 2.$$

  To avoid mistakes, place all exponents in parenthesis.
- Step 2 in the guidelines for solving exponential equations does NOT allow you to take the logarithm of each *term*. Remember that $\log_a(A + B) \neq \log_a A + \log_a B$.
- Be careful not to combine terms that are outside the logarithm with terms that are inside the logarithm. For example,

$$\log 2 + 5 \neq \log 7 \qquad \text{and} \qquad \frac{\log 4}{2} \neq \log 2.$$

- Be careful not to divide out terms that are inside two different logarithms. For example,

$$\frac{\log 15}{\log 3} \neq \log 5 \qquad \text{but} \qquad \log \frac{15}{3} = \log 5.$$

PROBLEMS

Solve for x in each of the following equations.

(NOTE: there are many different forms of the final answer.)

1. $3^{2x} = 7$

$$\begin{aligned} 3^{2x} &= 7 \\ \log 3^{2x} &= \log 7 \\ 2x \log 3 &= \log 7 \\ \frac{2x \log 3}{2 \log 3} &= \frac{\log 7}{2 \log 3} \\ x &= \frac{\log 7}{2 \log 3} \end{aligned}$$

$$\boxed{x = \frac{\log 7}{2 \log 3}}$$

2. $e^{4-3x} = 2$

Here, we will use the natural logarithm. Remember that $\ln e = 1$.

$$\begin{aligned} e^{4-3x} &= 2 \\ \ln e^{4-3x} &= \ln 2 \\ (4-3x) \ln e &= \ln 2 \\ 4 - 3x &= \ln 2 \\ -3x &= -4 + \ln 2 \\ x &= \frac{-4 + \ln 2}{-3} \\ x &= \frac{4 - \ln 2}{3} \end{aligned}$$

$$\boxed{x = \frac{4 - \ln 2}{3}}$$

3. $5^{2x-1} = 4$

$$\begin{aligned} 5^{2x-1} &= 4 \\ \log 5^{2x-1} &= \log 4 \\ (2x-1) \log 5 &= \log 4 \\ 2x \log 5 - \log 5 &= \log 4 \\ 2x \log 5 &= \log 4 + \log 5 \\ x &= \frac{\log 4 + \log 5}{2 \log 5} \end{aligned}$$

$$\boxed{x = \frac{\log 4 + \log 5}{2 \log 5}}$$

4. $3^x = 4^{x+1}$

$$\begin{aligned} 3^x &= 4^{x+1} \\ \log 3^x &= \log 4^{x+1} \\ x \log 3 &= (x+1) \log 4 \\ x \log 3 &= x \log 4 + \log 4 \\ x \log 3 - x \log 4 &= \log 4 \\ x(\log 3 - \log 4) &= \log 4 \\ x &= \frac{\log 4}{\log 3 - \log 4} \end{aligned}$$

$$\boxed{x = \frac{\log 4}{\log 3 - \log 4}}$$

5. $7 + 2^{x+4} = 12$

$$\begin{aligned} 7 + 2^{x+4} &= 12 \\ 2^{x+4} &= 5 \\ \log 2^{x+4} &= \log 5 \\ (x+4)\log 2 &= \log 5 \\ x\log 2 + 4\log 2 &= \log 5 \\ x \log 2 &= \log 5 - 4\log 2 \\ x &= \frac{\log 5 - 4\log 2}{\log 2} \end{aligned}$$

$$\boxed{x = \frac{\log 5 - 4\log 2}{\log 2}}$$

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6. $e^{2x} - 5e^x + 6 = 0$

Here we cannot isolate the exponential expression. So, we will factor the expression.

$$\begin{aligned} e^{2x} - 5e^x + 6 &= 0 \\ (e^x - 3)(e^x - 2) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} e^x - 3 &= 0 \\ e^x &= 3 \\ \ln e^x &= \ln 3 \\ x \ln e &= \ln 3 \\ x &= \ln 3 \end{aligned} \qquad \begin{aligned} e^x - 2 &= 0 \\ e^x &= 2 \\ \ln e^x &= \ln 2 \\ x \ln e &= \ln 2 \\ x &= \ln 2 \end{aligned}$$

$$\boxed{x = \ln 3, \qquad x = \ln 2}$$

7. $e^{4x} + 7e^{2x} - 18 = 0$

Once again, we will factor the expression.

$$\begin{aligned} e^{4x} + 7e^{2x} - 18 &= 0 \\ (e^{2x} + 9)(e^{2x} - 2) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} e^{2x} + 9 &= 0 \\ e^{2x} &\neq -9 \end{aligned} \qquad \begin{aligned} e^{2x} - 2 &= 0 \\ e^{2x} &= 2 \\ \ln e^{2x} &= \ln 2 \\ 2x \ln e &= \ln 2 \\ 2x &= \ln 2 \\ x &= \frac{\ln 2}{2} \end{aligned}$$

$$\boxed{x = \frac{\ln 2}{2}}$$

8. $4e^{3x} - 8e^{7x} = 0$

$$4e^{3x} - 8e^{7x} = 0$$

$$4e^{3x}(1 - 2e^{4x}) = 0$$

Setting each factor equal to zero, we get

$$4e^{3x} = 0$$

$$e^{3x} \neq 0$$

$$1 - 2e^{4x} = 0$$

$$-2e^{4x} = -1$$

$$e^{4x} = \frac{1}{2}$$

$$\ln e^{4x} = \ln \frac{1}{2}$$

$$4x \ln e = \ln \frac{1}{2}$$

$$4x = \ln \frac{1}{2}$$

$$x = \frac{1}{4} \ln \frac{1}{2}$$

$$\boxed{x = \frac{1}{4} \ln \frac{1}{2}}$$

9. $3\left(e^{4x-5} + 2\right) = 15$

$$3\left(e^{4x-5} + 2\right) = 15$$

$$e^{4x-5} + 2 = 5$$

$$e^{4x-5} = 3$$

$$\ln e^{4x-5} = \ln 3$$

$$(4x - 5) \ln e = \ln 3$$

$$4x - 5 = \ln 3$$

$$4x = 5 + \ln 3$$

$$x = \frac{5 + \ln 3}{4}$$

$$\boxed{x = \frac{5 + \ln 3}{4}}$$