Definition:

- ***n*-th root of *a***: The n-th root of a, denoted $\sqrt[n]{a}$, is a number whose n-th power equals a. In other words,

$$\sqrt[n]{a} = b \quad \text{means} \quad b^n = a.$$

The number n is called the **index**.

Important Properties:

- **Product rule for radicals**: If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and n is a positive integer,

$$\boxed{\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.}$$

In other words, the product of radicals is the radical of the product.

- **FOIL:** First Outer Inner Last. This is one method for multiplying factors which have two terms.
- **Distributive Property:** Recall that

$$a(b + c) = ab + ac \qquad \text{and} \qquad a(b - c) = ab - ac$$

- Special Formulas: Note that you do not need to memorize these formulas. They arise by using FOIL.

$$\boxed{(x + y)^2 = x^2 + 2xy + y^2}$$

$$\boxed{(x - y)^2 = x^2 - 2xy + y^2}$$

$$\boxed{(x - y)(x + y) = x^2 - y^2}$$

Simplifying radicals: A radical is in simplest form when the following conditions are satisfied.

- The quantity under the radical has no factor raised to a power greater than or equal to the index.
- There is no fraction under the radical.
- There is no radical in the denominator.
- There is no common factor, other than 1, between the exponents on factors under the radical and the index.

Common Mistakes to Avoid:

- Do not drop the index when working with cubes and other n-th roots.
- Do NOT distribute factors on the outside of the radical inside the radical. For example,

$$3x\sqrt{2x} \neq \sqrt{6x^2}.$$

- Remember $(a + b)^2 \neq a^2 + b^2$.

PROBLEMS

Multiply each product. Write all answers in simplest form.

1. $3\sqrt{2}\left(5\sqrt{3}-4\right)$

$$3\sqrt{2}\left(5\sqrt{3}-4\right)$$
$$\boxed{15\sqrt{6}-12\sqrt{2}}$$

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2. $5x\sqrt[3]{9}\left(2\sqrt[3]{3}-6\sqrt[3]{9}\right)$

$$5x\sqrt[3]{9}\left(2\sqrt[3]{3}-6\sqrt[3]{9}\right)$$
$$10x\sqrt[3]{27}-30x\sqrt[3]{81}$$
$$10x\cdot 3-30x\sqrt[3]{27\cdot 3}$$
$$30x-30x\sqrt[3]{27}\sqrt[3]{3}$$
$$\boxed{30x-90x\sqrt[3]{3}}$$

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3. $\left(3\sqrt{5}+2\right)\left(2\sqrt{5}-7\right)$

$$\left(3\sqrt{5}+2\right)\left(2\sqrt{5}-7\right)$$
$$6\sqrt{25}-21\sqrt{5}+4\sqrt{5}-14$$
$$6\cdot 5-21\sqrt{5}+4\sqrt{5}-14$$
$$30-21\sqrt{5}+4\sqrt{5}-14$$
$$\boxed{16-17\sqrt{5}}$$

4. $\left(\sqrt{8}-3\right)^2$

$$\left(\sqrt{8}-3\right)^2$$
$$\left(\sqrt{8}-3\right)\left(\sqrt{8}-3\right)$$
$$\sqrt{64}-3\sqrt{8}-3\sqrt{8}+9$$
$$8-3\sqrt{8}-3\sqrt{8}+9$$
$$17-6\sqrt{8}$$
$$17-6\sqrt{4\cdot 2}$$
$$17-6\sqrt{4}\sqrt{2}$$
$$17-6\cdot 2\sqrt{2}$$
$$\boxed{17-12\sqrt{2}}$$

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5. $\left(2\sqrt{3}+\sqrt{6}\right)^2$

$$\left(2\sqrt{3}+\sqrt{6}\right)^2$$
$$\left(2\sqrt{3}+\sqrt{6}\right)\left(2\sqrt{3}+\sqrt{6}\right)$$
$$4\sqrt{9}+2\sqrt{18}+2\sqrt{18}+\sqrt{36}$$
$$4\cdot 3+4\sqrt{18}+6$$
$$12+4\sqrt{9}\sqrt{2}+6$$
$$18+4\cdot 3\sqrt{2}$$
$$\boxed{18+12\sqrt{2}}$$

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6. $\left(3\sqrt{5}-4\right)\left(3\sqrt{5}+4\right)$

$$\left(3\sqrt{5}-4\right)\left(3\sqrt{5}+4\right)$$
$$9\sqrt{25}+12\sqrt{5}-12\sqrt{5}-16$$
$$9\cdot 5-16$$
$$45-16$$
$$\boxed{29}$$

7. $\left(\sqrt{7}+2\right)\left(\sqrt{2}-4\right)$

$$\left(\sqrt{7}+2\right)\left(\sqrt{2}-4\right)$$
$$\boxed{\sqrt{14}-4\sqrt{7}+2\sqrt{2}-8}$$

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8. $\left(2\sqrt{8}-\sqrt{3}\right)\left(2\sqrt{48}-\sqrt{2}\right)$

$$\left(2\sqrt{8}-\sqrt{3}\right)\left(2\sqrt{48}-\sqrt{2}\right)$$
$$\left(2\sqrt{4\cdot 2}-\sqrt{3}\right)\left(2\sqrt{16\cdot 3}-\sqrt{2}\right)$$
$$\left(2\cdot 2\sqrt{2}-\sqrt{3}\right)\left(2\cdot 4\sqrt{3}-\sqrt{2}\right)$$
$$\left(4\sqrt{2}-\sqrt{3}\right)\left(8\sqrt{3}-\sqrt{2}\right)$$
$$32\sqrt{6}-4\sqrt{4}-8\sqrt{9}+\sqrt{6}$$
$$32\sqrt{6}-4\cdot 2-8\cdot 3+\sqrt{6}$$
$$32\sqrt{6}-8-24+\sqrt{6}$$
$$\boxed{-32+33\sqrt{6}}$$

NOTE: You could also have multiplied using FOIL first and then simplified your answer.