Important Properties:

- **Difference of squares**:
$$\boxed{a^2 - b^2 = (a - b)(a + b)}$$
- **Difference of cubes**:
$$\boxed{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$$
- **Sum of cubes**:
$$\boxed{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$$
- Remember that you can always check your factoring by multiplication.
- Always look for the greatest common factor (GCF) first.
- Remember that an expression containing four terms will most likely factor by grouping.

Common Mistakes to Avoid:

- Remember to factor out the variable to its *smallest* exponent.
- Do NOT forget to carry your GCF all the way to the end of the problem.

PROBLEMS

Factor completely.

1. $x^{1/2} - x^{5/2}$

$$x^{1/2} - x^{5/2}$$
$$x^{1/2}\left(1 - x^2\right)$$
$$\boxed{x^{1/2}(1 - x)(1 + x)}$$

2. $x^{-1/2}(x + 3)^{1/2} + x^{1/2}(x + 3)^{-1/2}$

$$x^{-1/2}(x + 3)^{1/2} + x^{1/2}(x + 3)^{-1/2}$$
$$x^{-1/2}(x + 3)^{-1/2}\left[(x + 3) + x\right]$$
$$\boxed{x^{-1/2}(x + 3)^{-1/2}(2x + 3)}$$

3. $x^{3/2} + 8x^{1/2} + 15x^{-1/2}$

$$x^{3/2} + 8x^{1/2} + 15x^{-1/2}$$
$$x^{-1/2}(x^2 + 8x + 15)$$
$$\boxed{x^{-1/2}(x+3)(x+5)}$$

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4. $2x^{1/2} + 5x^{-1/2} + 2x^{-3/2}$

$$2x^{1/2} + 5x^{-1/2} + 2x^{-3/2}$$
$$x^{-3/2}(2x^2 + 5x + 2)$$
$$\boxed{x^{-3/2}(2x+1)(x+2)}$$

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5. $(x+2)^{7/2} - (x+2)^{3/2}$

$$(x+2)^{7/2} - (x+2)^{3/2}$$
$$(x+2)^{3/2}\left[(x+2)^2 - 1\right]$$
$$(x+2)^{3/2}\left(x^2 + 4x + 4 - 1\right)$$
$$(x+2)^2(x^2 + 4x + 3)$$
$$\boxed{(x+2)^2(x+1)(x+3)}$$

OR

$$(x+2)^{7/2} - (x+2)^{3/2}$$
$$(x+2)^{3/2}\left[(x+2)^2 - 1\right]$$
$$(x+2)^2\left[(x+2) - 1\right]\left[(x+2) + 1\right]$$
$$(x+2)^2(x+2-1)(x+2+1)$$
$$\boxed{(x+2)^2(x+1)(x+3)}$$

6. $4(x+1)^{1/2} - 5(x+1)^{3/2} + (x+1)^{5/2}$

$$4(x+1)^{1/2} - 5(x+1)^{3/2} + (x+1)^{5/2}$$
$$(x+1)^{1/2}\left[4 - 5(x+1) + (x+1)^2\right]$$
$$(x+1)^{1/2}\left[4 - 5x - 5 + x^2 + 2x + 1\right]$$
$$(x+1)^{1/2}(x^2 - 3x)$$
$$\boxed{x(x+1)^{1/2}(x-3)}$$

OR

$$4(x+1)^{1/2} - 5(x+1)^{3/2} + (x+1)^{5/2}$$
$$(x+1)^{1/2}\left[4 - 5(x+1) + (x+1)^2\right]$$
$$(x+1)^{1/2}\left[(x+1)^2 - 5(x+1) + 4\right]$$
$$(x+1)^{1/2}\left[(x+1) - 1\right]\left[(x+1) - 4\right]$$
$$(x+1)^{1/2}(x+1-1)(x+1-4)$$
$$\boxed{x(x+1)^{1/2}(x-3)}$$

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7. $x^{1/2} - 3x^{1/3} - 3x^{1/6} - 9$

$$\underbrace{x^{1/2} - 3x^{1/3}} - \underbrace{3x^{1/6} - 9}$$
$$x^{1/3}\left(x^{1/6} - 3\right) - 3(x^{1/6} - 3)$$
$$\boxed{\left(x^{1/6} - 3\right)\left(x^{1/3} - 3\right)}$$

8. $12x^{4/3} - 5x^{2/3} - 2$

$$12x^{4/3} - 5x^{2/3} - 2$$
$$\boxed{(3x^{2/3} - 2)(4x^{2/3} + 1)}$$

OR

Letting $u = x^{2/3}$

$$12x^{4/3} - 5x^{2/3} - 2$$
$$12u^2 - 5u - 2$$
$$(3u - 2)(4u + 1)$$
$$\boxed{(3x^{2/3} - 2)(4x^{2/3} + 1)}$$

9. $x^{3/5} + 5x^{2/5} - 3x^{1/5} - 15$

$$\underbrace{x^{3/5} + 5x^{2/5}} - \underbrace{3x^{1/5} - 15}$$
$$x^{2/5}(x^{1/5} + 5) - 3(x^{1/5} + 5)$$
$$\boxed{(x^{1/5} + 5)(x^{2/5} - 3)}$$