SOLVING QUADRATIC EQUATIONS BY THE QUADRATIC FORMULA

Definition:

- **Quadratic Equation**: is an equation that can be written in the form

$$ax^2 + bx + c = 0,$$

where a, b, and c are real numbers, $a \neq 0$.

Important Properties:

- **Quadratic Formula**: The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.}$$

- The quadratic formula is a result of solving $ax^2 + bx + c = 0$ by completing the square.
- The quadratic formula can be used to solve *any* quadratic equation.
- If $b^2 - 4ac < 0$ then there are no real solutions to the quadratic equation.
- If $b^2 - 4ac = 0$, then the quadratic equation has only one real zero.
- If $b^2 - 4ac > 0$, then the quadratic equation has two real solutions.

Common Mistakes to Avoid:

- Before identifying a, b, and c to be used in the quadratic formula, make sure one side of your equation is zero.
- In the quadratic formula, the $-b$ is also divided by $2a$.
- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$.
- Be careful when simplifying your final answer. Remember that

$$\frac{b + ac}{c} \neq b + a, \qquad \text{and} \qquad \frac{c + a}{c} \neq 1 + a.$$

- The quadratic formula can only be used on a quadratic equation. Do not use on a quadratic-type equation without a change of variables.

PROBLEMS

Solve the following equations using the quadratic formula.

1. $6x^2 - 5x - 4 = 0$

Note, $a = 6,\ b = -5$, and $c = -4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(6)(-4)}}{2(6)}$$

$$x = \frac{5 \pm \sqrt{25 + 96}}{12}$$

$$x = \frac{5 \pm \sqrt{121}}{12}$$

$$x = \frac{5 \pm 11}{12}$$

$$x = \frac{5 + 11}{12} = \frac{16}{12} = \frac{4}{3},$$

$$x = \frac{5 - 11}{12} = \frac{-6}{12} = -\frac{1}{2}$$

$$\boxed{x = \frac{4}{3}, \quad x = -\frac{1}{2}}$$

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2. $x^2 + 3x - 2 = 0$

Note, $a = 1,\ b = 3$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 8}}{2}$$

$$x = \frac{-3 \pm \sqrt{17}}{2}$$

$$\boxed{x = \frac{-3 \pm \sqrt{17}}{2}}$$

3. $x^2 - 4 = 2x$

We must make one side zero before we can identify a, b, and c.

$$x^2 - 4 = 2x$$
$$x^2 - 2x - 4 = 0$$

Therefore, $a = 1,\ b = -2$, and $c = -4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2}$$

$$x = \frac{2 \pm \sqrt{20}}{2}$$

$$x = \frac{2 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2(1 \pm \sqrt{5})}{2}$$

$$x = 1 \pm \sqrt{5}$$

$$\boxed{x = 1 \pm \sqrt{5}}$$

4. $4x^2 + 4x - 1 = 0$

Note, $a = 4$, $b = 4$, and $c = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(4)(-1)}}{2(4)}$$
$$x = \frac{-4 \pm \sqrt{16 + 16}}{8}$$
$$x = \frac{-4 \pm \sqrt{32}}{8}$$
$$x = \frac{-4 \pm 4\sqrt{2}}{8}$$
$$x = \frac{4(-1 \pm \sqrt{2})}{8}$$
$$x = \frac{-1 \pm \sqrt{2}}{2}$$
$$\boxed{x = \frac{1 \pm \sqrt{2}}{2}}$$

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5. $5x^2 - 3x = 7$

$$5x^2 - 3x = 7$$
$$5x^2 - 3x - 7 = 0$$

Therefore, $a = 5$, $b = -3$, and $c = -7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-7)}}{2(5)}$$
$$x = \frac{3 \pm \sqrt{9 + 140}}{10}$$
$$x = \frac{3 \pm \sqrt{149}}{10}$$
$$\boxed{x = \frac{3 \pm \sqrt{149}}{10}}$$

6. $-2x(x + 4) = -6$

$$-2x(x + 4) = -6$$
$$-2x^2 - 8x = -6$$
$$-2x^2 - 8x + 6 = 0$$

Note, that a -2 can be divided out of each term. Therefore, before using the quadratic formula, we will divide each term by -2.

$$-2x^2 - 8x + 6 = 0$$
$$x^2 + 4x - 3 = 0$$

Hence, $a = 1$, $b = 4$, and $c = -3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{16 + 12}}{2}$$
$$x = \frac{-4 \pm \sqrt{28}}{2}$$
$$x = \frac{-4 \pm 2\sqrt{7}}{2}$$
$$x = \frac{2(-2 \pm \sqrt{7})}{2}$$
$$x = -2 \pm \sqrt{7}$$
$$\boxed{x = -2 \pm \sqrt{7}}$$

7. $7x^2 - 3x = -6$

We must first make one side zero before identifying a, b, and c.

$$7x^2 - 3x = -6$$
$$7x^2 - 3x + 6 = 0$$

Therefore, $a = 7$, $b = -3$, and $c = 6$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(7)(6)}}{2(7)}$$
$$x = \frac{3 \pm \sqrt{9 - 168}}{14}$$
$$x = \frac{3 \pm \sqrt{-159}}{14}$$

$$\boxed{\text{No real solution}}$$

8. $x^2 + 16 = 8x$

Once again, we must have one side zero before we can identify a, b, or c.

$$x^2 + 16 = 8x$$
$$x^2 - 8x + 16 = 0$$

Therefore, $a = 1$, $b = -8$, and $c = 16$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}$$
$$x = \frac{8 \pm \sqrt{64 - 64}}{2}$$
$$x = \frac{8}{2}$$
$$x = 4$$

$$\boxed{x = 4}$$