Definition:

- **Prime**: We say that a polynomial is prime if it cannot be factored.

Important Properties:

- Difference of Squares:
$$\boxed{x^2 - y^2 = (x - y)(x + y)}$$
- Difference of Cubes:
$$\boxed{x^3 - y^3 = (x - y)(x^2 + xy + y^2)}$$
- Sum of Cubes:
$$\boxed{x^3 + y^3 = (x + y)(x^2 - xy + y^2)}$$
- Remember that it does not matter in what order you list the factors. For example,
$$(2x - 1)(x + 2) = (x + 2)(2x - 1).$$

Common Mistakes to Avoid:

- The sum of two cubes does factor but the sum of two squares does NOT.
- $x^3 + y^3 \neq (x + y)^3$ AND $x^3 - y^3 \neq (x - 3)^2$.

PROBLEMS

Factor completely.

1. $9x^2 - 4$

$$9x^2 - 4$$
$$(3x)^2 - 2^2$$
$$\boxed{(3x - 2)(3x + 2)}$$

2. $16x^2 - 25$

$$16x^2 - 25$$
$$(4x)^2 - 5^2$$
$$\boxed{(4x - 5)(4x + 5)}$$

3. $16x^2 + 9$

$$16x^2 + 9$$
$$\boxed{\text{PRIME}}$$

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4. $81x^2 - 49$

$$81x^2 - 49$$
$$(9x)^2 - 7^2$$
$$\boxed{(9x - 7)(9x + 7)}$$

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5. $16x^2 - y^2$

$$16x^2 - y^2$$
$$(4x)^2 - y^2$$
$$\boxed{(4x - y)(4x + y)}$$

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6. $x^3 - 8$

$$x^3 - 8$$
$$x^3 - 2^3$$
$$\boxed{(x - 2)(x^2 + 2x + 4)}$$

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7. $x^3 - 27$

$$x^3 - 27$$
$$x^3 - 3^3$$
$$\boxed{(x - 3)(x^2 + 3x + 9)}$$

8. $x^3 + 8$

$$x^3 + 8$$
$$x^3 + 2^3$$
$$\boxed{(x + 2)(x^2 - 2x + 4)}$$

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9. $x^3 - 1$

$$x^3 - 1$$
$$\boxed{(x - 1)(x^2 + x + 1)}$$

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10. $8x^3 - 1$

$$8x^3 - 1$$
$$(2x)^3 - 1^3$$
$$(2x - 1)((2x)^2 + 2x + 1)$$
$$\boxed{(2x - 1)(4x^2 + 2x + 1)}$$

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11. $27x^3 + 1$

$$27x^3 + 1$$
$$(3x)^3 + 1^3$$
$$(3x + 1)((3x)^2 - 3x + 1)$$
$$\boxed{(3x + 1)(9x^2 - 3x + 1)}$$

12. $64x^3 - y^3$

$$64x^3 - y^3$$
$$(4x)^3 - y^3$$
$$(4x - y)((4x)^2 + 4xy + y^2)$$
$$\boxed{(4x - y)(16x^2 + 4xy + y^2)}$$

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13. $16x^4 - 81$

$$16x^4 - 81$$
$$(4x^2 - 9)(4x^2 + 9)$$
$$\boxed{(2x - 3)(2x + 3)(4x^2 + 9)}$$

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14. $81x^4 - 1$

$$81x^4 - 1$$
$$(9x^2)^2 - 1$$
$$(9x^2 - 1)(9x^2 + 1)$$
$$\boxed{(3x - 1)(3x + 1)(9x^2 + 1)}$$