Transformations of functions:

- **Vertical Shift**: Suppose that $c > 0$.
  - The equation $y = f(x) + c$ shifts the graph of $y = f(x)$ up c units. (Adding a constant on the outside of a functions shifts the graph up.)
  - The equation $y = f(x) - c$ shifts the graph of $y = f(x)$ down c units. (Subtracting a constant on the outside of a function shifts the graph down.)
- **Horizontal Shift**: Suppose that $c > 0$.
  - The equation $y = f(x + c)$ shifts the graph of $y = f(x)$ to the left c units. (Adding a constant inside the function shifts the graph left.)
  - The equation $y = f(x - c)$ shifts the graph of $y = f(x)$ to the right c units. (Subtracting a constant inside the function shifts the graph right.)
- **Reflections**:
  - The equation $y = -f(x)$ reflects the graph of $y = f(x)$ with respect to the $x-$axis. (Multiplying by a negative on the outside of a function flips the graph with respect to the $x-$axis.)
  - The equation $y = f(-x)$ reflects the graph of $y = f(x)$ with respect to the the $y-$axis. (Multiplying by a negative inside the function flips the graph with respect to the $y-$axis.)
- **Vertical Stretching and Shrinking**:
  - When $c > 1$, the equation $y = cf(x)$ stretches the graph of $y = f(x)$ vertically by a factor of c. (Multiplying by a number, larger than one, on the outside of a function causes the graph to be stretched or narrowed by a factor of c.)
  - When $0 < c < 1$, the equation $y = cf(x)$ shrinks the graph of $y = f(x)$ vertically by a factor of c. (Multiplying by a number, between zero and one, on the outside of a functions causes the graph to shrink or widen by a factor of c.)

Important Properties:

- When combining the transformations, first identify how every number affects the graph of f.

Common Mistakes to Avoid:

- Errors frequently occur with horizontal shifts. Remember if we add a constant inside a function, we shift left; if we subtract a constant inside a function, we shift right.
- When multiplying a function by a negative number, say $-c$, remember that the negative is a flip and the c is either a stretch or shrink (depending on its value).

PROBLEMS

1. **Explain how the graph of g is obtained from the graph of f. Be specific!**

(a) $f(x) = x^2$; $g(x) = (x-4)^2$

Since we are subtracting a constant inside the function, we need to shift the the graph of f right 4 units.

$\boxed{\text{Shift right 4 units}}$

(b) $f(x) = x^2$; $g(x) = x^2 - 5$

Since we are subtracting a constant on the outside of the function, we need to shift the graph of f down 5 units.

$\boxed{\text{Shift down 5 units}}$

(c) $f(x) = \sqrt{x}$; $g(x) = 3\sqrt{x+1}$

Here, we have two transformations. First, since we are multiplying by 3 on the outside this is a vertical stretch by a factor of 3. Adding 1 inside the function causes a shift of 1 unit to the left.

$\boxed{\text{Shift left 1 unit, stretch by a factor of 3}}$

(d) $f(x) = x^3$; $g(x) = \frac{1}{4}(x-5)^3 + 2$

Now we have three transformations. The multiplication of $\frac{1}{4}$ on the outside of the function is a vertical shrink by a factor of $\frac{1}{4}$. Subtracting 5 inside the function causes a shift right 5 units. Finally, adding 2 outside the function means moving the graph up 2 units.

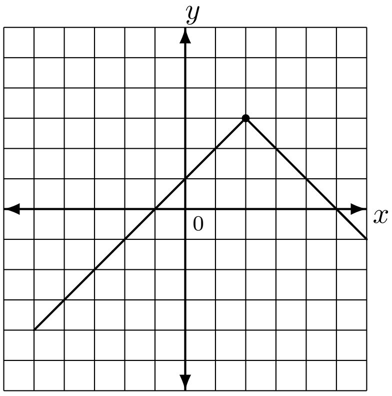
$\boxed{\text{Shift right 5, up 2, shrink by factor of } \dfrac{1}{4}}$

(e) $f(x) = \sqrt[3]{x}$; $g(x) = -4\sqrt[3]{x-2} + 6$

Here we have four transformations. The multiplication of -4 on the outside of the function is a vertical stretch by a factor of 4 and a flip with respect to the $x-$axis. Adding 6 on the outside of the function is a shift up 6 units. Finally, subtracting 2 inside the function means moving the graph right 2 units.

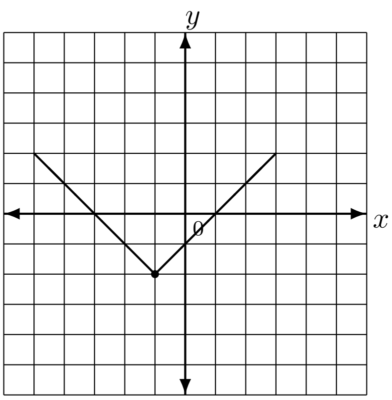
$\boxed{\text{Shift right 2, up 6, stretch, flip x-axis}}$

2. **The following functions are transformations of $y = |x|$. Determine each function's equation.**



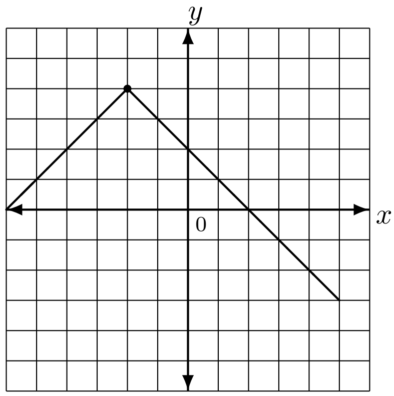
This graph has been shifted right 2 units (subtracting inside the function); shifted up 3 units (adding outside the function); and reflected or flipped across the $x-$axis (multiplying a negative outside the function).

$$\boxed{f(x) = -|x-2| + 3}$$



The graph has been shifted left 1 unit (adding inside the function); and shifted down 2 units (subtracting outside the function)

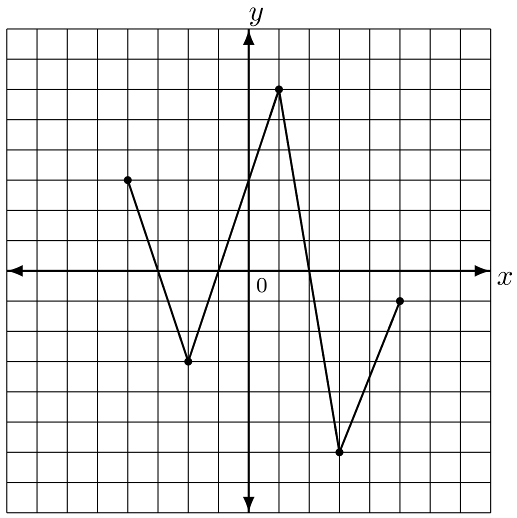
$$\boxed{f(x) = |x+1| - 2}$$



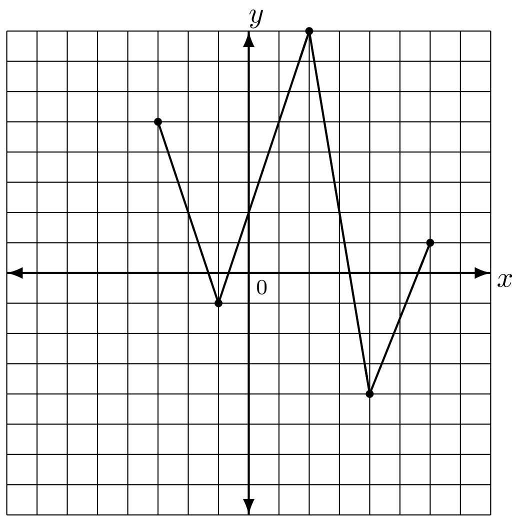
The graph has been shifted left 2 units (adding inside the function); shifted up 4 units (adding outside the function); and reflected across the $x-$axis.

$$\boxed{f(x) = -|x+2| + 4}$$

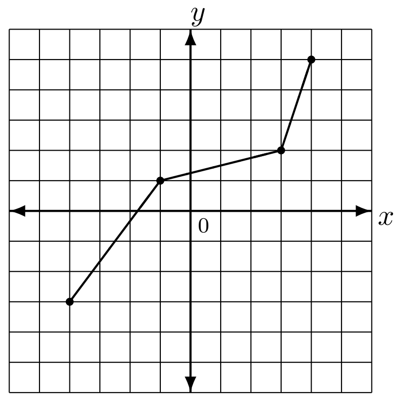
3. **The graph of $y = f(x)$ is given below. Sketch the graph of $y = f(x-1) + 2$.**



The equation $y = f(x-1) + 2$ will shift the graph to the right 1 unit and shift it up 2 units. The answer is given below.



4. **The graph of $y = f(x)$ is given. Sketch the graph of $y = -f(x) + 3$.**



For this graph we need to reflect the graph with respect to the $x-$axis and then shift it up 3 units. The answer is given below.