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Hyperbolic monopole in a smectic C liquid crystal

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A new monopole structure consisting of a combination of a hyperbolic "hedgehog" and two disclinations, each of strength $m = 1$, has been discovered in a smectic C liquid crystal. As the temperature is changed, there is a phase transition from a hyperbolic monopole to a radial monopole. This transition is unrelated to a change in the phase state of the smectic C crystal or to conditions at the boundary.

Certain special defects, which have been found to exist[1] in a smectic C liquid crystal, have been called "monopoles" because of their structural similarity to a Dirac monopole. They consist of pairs of disclinations of strength $m_1 = m_2 = 1$ which emerge from a radial point defect ("hedgehog") in the distribution of the field of the normal ($\mathbf{n}$) to the layers of the liquid crystal:

$$\mathbf{n}(x, y, z) = (x, y, z)/(x^2 + y^2 + z^2)^{1/2}. \tag{1}$$

In the present letter we report the experimental observation of a monopole in which the point defect has the shape of a hyperbolic hedgehog,

$$\mathbf{n}(x, y, z) = (-x, -y, z)/(x^2 + y^2 + z^2)^{1/2}, \tag{2}$$

and the associated disclinations are of strength $m_3 = m_4 = -1$. A phase transition occurs between the two types of monopoles as the temperature is varied.

The new structures were observed in freely suspended drops of the heptyloxyphenyl ester of octyloxybenzoic acid, which has a smectic C phase in the temperature interval 55–69 °C. The drops are prepared by the technique described in Ref. 1 and have a radius $R \approx 10\ \mu$m.

The textures of the drops are characterized by the presence of four defect elements [Fig. 1, (a)–(d)]: a ring A, $d \approx 5\ \mu$m in diameter in the volume; a pair of
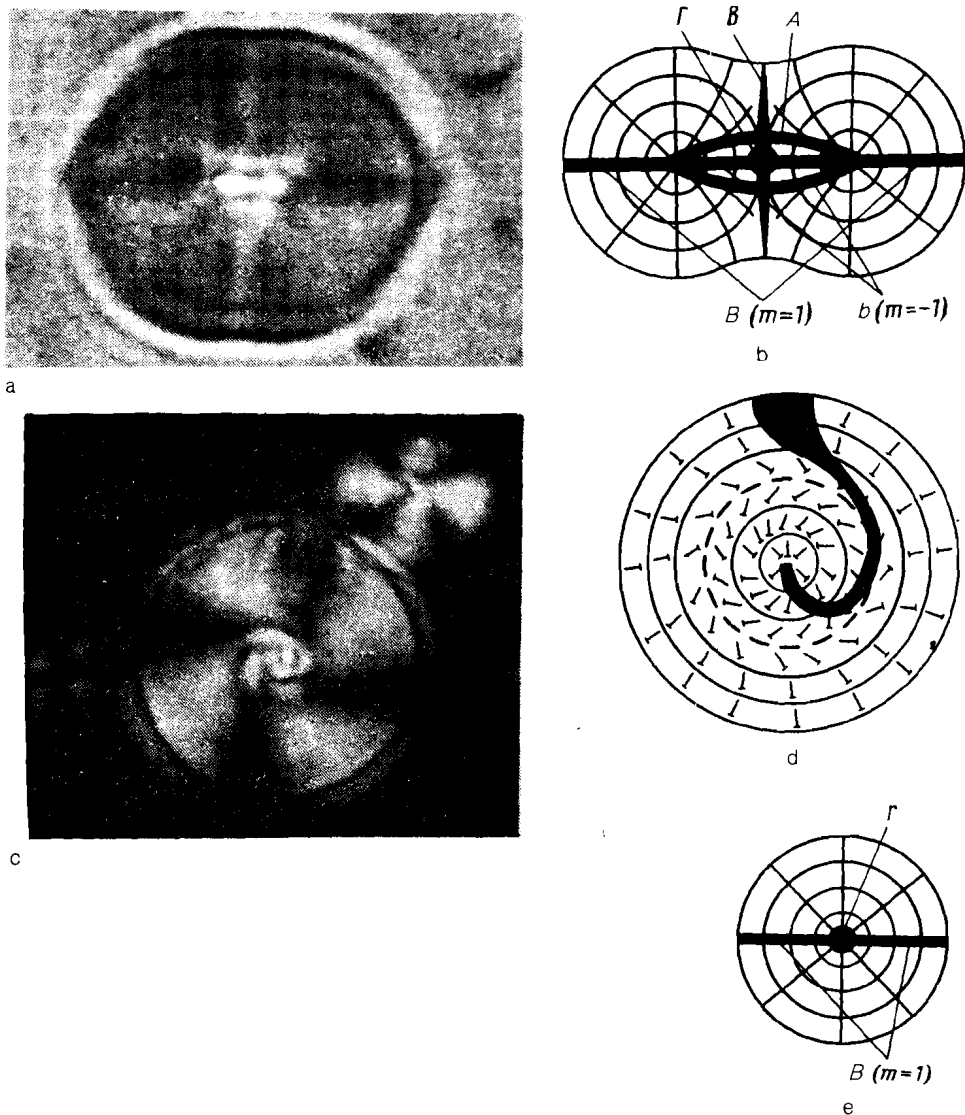
FIG. 1. Hyperbolic (a–d) and radial (e) monopoles in a drop of a smectic C liquid crystal. a,c—Photomicrographs of drops for two mutually perpendicular orientations; b,d—corresponding distributions of the smectic layers and molecules. Also shown in part d is one of the four extinction branches observed in the photomicrograph in part c. Also shown in part b are the distribution of the field of the normal ($\mathbf{n}$) to the layers of the liquid crystal and the arrangement of defects in the drop. For clarity, the ring A has been taken out of the equatorial plane.

disclinations B; an axial line B, perpendicular to the plane of ring A; and, finally, a point defect Γ at the center of the drop. Observation of the arrangement of extinction branches and isochromic lines under a microscope with crossed Nicol prisms reveals that the layers of the liquid crystal at the periphery of a drop produce the shape of the

boundary of the drop, which is reminiscent of an oblate ellipsoid of revolution, while near the center of the drop the layers appear as toric surfaces [Fig. 1, (b) and (d)]. Ring A, which lies along the axis of the torus, is a singular disclination of strength $m = 1$ in the n field; the distribution of molecules on the ring (except for the two intersections A and B) remains regular because of a smooth transition to an orientation along the axis of the defect. In other words, a disclination of strength $m = 1$ "pours out" into a third dimension in the system of smectic layers [Fig. 1(d)]. Experimentally, this pouring out is seen as a twisting of the extinction branches during observation with crossed Nicol prisms behind a drop which is oriented in such a fashion that the ring A is oriented perpendicular to the optical axis of the microscope [cf. Figs. 1(c) and 1(d)].

Line B is not singular in either the distribution of the field n or the distribution of molecules. It stems from slight bends (hollows) of the layers of the liquid crystal and from inhomogeneities in the inclinations of the molecules in the layers as a result of these bends [Fig. 1(b)]. This interpretation is supported by the circumstance that line B becomes narrower and smoother as it approaches the periphery of the drop [Fig. 1(a)].

The field of the normal (n) to the smectic layers for the configuration as a whole is thus close to the distribution of a radial hedgehog (1) at distance)s greater than $2d$ from the center of the drop and a hyperbolic hedgehog (2) at distances less than $2d$ [Fig. 1(b)].

Of particular interest for this discussion is the pair of disclinations B which are oriented perpendicular to the layers of the liquid crystal and which are therefore singularities in the field ($\vec{\tau}$) of projections of the long axes of the molecules onto the surface of the layers. Actually, each line of this pair consists of two series-connected disclinations, one of strength $m = 1$ and one of strength $m = -1$. At distances greater than $2d$, the layers are topologically equivalent to spheres; according to the Poincaré theorem, each layer contains two singular points of the field $\vec{\tau}$ with indices[1] $m_1 = m_2 = 1$. These points form a pair of disclinations of strength $m_1 = m_2 = 1$. For layers of toric configuration, the sum index of all the singular points of the field $\vec{\tau}$ must be zero by virtue of the Poincaré theorem. It can be seen from Fig. 1(a) that disclinations with $m_1 = m_2 = 1$ reach the very core of ring A, so that a pair of singular points with indices $m_3 = m_4 = -1$ must exist on each toric layer and compensate for the sum index of the points $m_1 = m_2 = 1$ on the same layer. We thus see the formation of a pair of dislocations of strength $m_3 = m_4 = -1$, which connect ring A with the center of hyperbolic hedgehog Γ. From the standpoint of homotopic classification,[2] lines B and A belong to the same class $(0,a^2)$ of the fundamental degeneracy space group of the smectic C liquid crystal, and the relation $(0,a^2) = (0,a^2)^3$ guarantees the possibility of this intersection of A with B.

The hyperbolic monopoles are stable in the medium studied at temperatures in the interval 55–66 °C. As the temperature is raised to 66 °C, the monopoles convert into radial monopoles [Fig. 1(e)] because the ring A contracts to a point. No change occurs in the orientation of the layers at the surface at this time. The transition is reversible and is of the nature of a second-order phase transition. The possibility of phase transitions of this sort between states of a liquid crystal differing only in the

symmetry of defects, and not accompanied by a change in the state of the medium itself, has been predicted for disclinations in nematic liquid crystals.[3] To describe the transition, we should introduce topological charges of the monopoles, which would have different values for hyperbolic and radial configurations of the field **n**. The order (N) of the mapping of the **n** field onto a unit sphere S^2 which has customarily been used cannot serve as this charge, since we have $N_{(1)} = N_{(2)} = 1$. It is convenient to choose a Morse index M: $M = 0$ for distribution (1) and $M = 2$ for distribution (2) (Ref. 4). The probable reason for the transition is a temperature dependence of the elastic constants of the liquid crystal. The mechanism for this transition deserves a detailed study.

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Translated by Dave Parsons