

# Trigonometric Formulas

## Half Angle Identities

$$\sin(\alpha/2) = \pm \sqrt{\frac{1 - \cos(\alpha)}{2}}$$

$$\cos(\alpha/2) = \pm \sqrt{\frac{1 + \cos(\alpha)}{2}}$$

$$\tan(\alpha/2) = \pm \sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}}$$

## Power Reducing Identities

$$\sin^2(u) = \frac{1 - \cos(2u)}{2}$$

$$\cos^2(u) = \frac{1 + \cos(2u)}{2}$$

$$\tan^2(u) = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

## Product to Sum Formulas

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = -\frac{1}{2}[\cos(u + v) - \cos(u - v)]$$

## Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2\left[\sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)\right]$$

$$\sin \alpha - \sin \beta = 2\left[\cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)\right]$$

$$\cos \alpha + \cos \beta = 2\left[\cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)\right]$$

$$\cos \alpha - \cos \beta = -2\left[\sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)\right]$$

Angle	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

## Quotient identities

$$\begin{aligned}\tan(\theta) &= \frac{\sin(\theta)}{\cos(\theta)} \\ \cot(\theta) &= \frac{\cos(\theta)}{\sin(\theta)}\end{aligned}$$

## Reciprocal identities

$$\begin{aligned}\sec(\theta) &= \frac{1}{\cos(\theta)} \\ \csc(\theta) &= \frac{1}{\sin(\theta)} \\ \cot(\theta) &= \frac{1}{\tan(\theta)}\end{aligned}$$

## Pythagorean identities

$$\begin{aligned}\sin^2(\theta) + \cos^2(\theta) &= 1 \\ 1 + \tan^2(\theta) &= \sec^2(\theta) \\ 1 + \cot^2(\theta) &= \csc^2(\theta)\end{aligned}$$

## Sum and Difference formulas

$$\begin{aligned}\sin(u + v) &= \sin(u) \cos(v) + \cos(u) \sin(v) \\ \sin(u - v) &= \sin(u) \cos(v) - \cos(u) \sin(v) \\ \cos(u + v) &= \cos(u) \cos(v) - \sin(u) \sin(v) \\ \cos(u - v) &= \cos(u) \cos(v) + \sin(u) \sin(v) \\ \tan(u + v) &= \frac{\tan(u) + \tan(v)}{1 - \tan(u) \tan(v)} \\ \tan(u - v) &= \frac{\tan(u) - \tan(v)}{1 + \tan(u) \tan(v)}\end{aligned}$$

## Double angle formulas

$$\begin{aligned}\sin(2\theta) &= 2 \sin(\theta) \cos(\theta) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2 \cos^2(\theta) - 1 \\ &= 1 - 2 \sin^2(\theta) \\ \tan(2\theta) &= \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}\end{aligned}$$