

Homework Assignment #6

Mathematics 41021 - *Theory of Matrices* – Spring 2007

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March 22, 2007 (These problems will be discussed in class on this day)

1. Use the Cayley-Hamilton Theorem to find the inverse of $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 2 \\ 2 & 1 & 2 \end{pmatrix}$.
2. Find $A^{520} + 3A^{70} - 7I$ for $A = \begin{pmatrix} -2 & 4 & 3 \\ 0 & 0 & 0 \\ -1 & 5 & 2 \end{pmatrix}$.
3. Let $A = \begin{pmatrix} 5 & 0 & 0 \\ 6 & a & 0 \\ 7 & 8 & b \end{pmatrix}$. Find a and b , if the trace of A is 5 and the product of eigenvalues is -5.
4. Let $A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Find a if the sum of the eigenvalues is 3.
5. What is wrong with this argument? ”

Since $p_A(t) = \det(tI - A)$, $p_A(A) = \det(AI - A) = \det(A - A) = \det(0) = 0$. Therefore, $p_A(A) = 0$.

6. Let $A \in M_n$ be a nonsingular matrix. Show that any matrix that commutes with A also commutes with A^{-1} .
7. Show that if the $A \in M_n$ has eigenvalues $\lambda_1, \dots, \lambda_n$, then $\sum_{i=1}^n \lambda_i^k = \text{tr}(A^k)$, for $k = 1, 2, \dots$.