Trigonometric Formulas

Double Angle Formulas

$$\begin{aligned}
\sin(2\theta) &= 2\sin\theta\cos\theta \\
\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\
&= 2\cos^2\theta - 1 \\
&= 1 - 2\sin^2\theta \\
\tan(2\theta) &= \frac{2\tan\theta}{1-\tan^2\theta}
\end{aligned}$$

Half Angle Formulas

$$\begin{aligned}
\sin(\frac{\alpha}{2}) &= \pm\sqrt{\frac{1-\cos\alpha}{2}} \\
\cos(\frac{\alpha}{2}) &= \pm\sqrt{\frac{1+\cos\alpha}{2}} \\
\tan(\frac{\alpha}{2}) &= \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}.
\end{aligned}$$

Power Reducing Identities

$$\begin{aligned}
\sin^2(u) &= \frac{1-\cos(2u)}{2} \\
\cos^2(u) &= \frac{1+\cos(2u)}{2} \\
\tan^2(u) &= \frac{1-\cos(2u)}{1+\cos(2u)}
\end{aligned}$$

Product to Sum Formulas

$$\begin{aligned}
\sin u\cos v &= \frac{1}{2}[\sin(u+v)+\sin(u-v)] \\
\cos u\sin v &= \frac{1}{2}[\sin(u+v)-\sin(u-v)] \\
\cos u\cos v &= \frac{1}{2}[\cos(u+v)+\cos(u-v)] \\
\sin u\sin v &= -\frac{1}{2}[\cos(u+v)-\cos(u-v)]
\end{aligned}$$

Sum to Product Formulas

$$\begin{aligned}
\sin\alpha+\sin\beta &= 2\sin(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2}) \\
\sin\alpha-\sin\beta &= 2\cos(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2}) \\
\cos\alpha+\cos\beta &= 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2}) \\
\cos\alpha-\cos\beta &= -2\sin(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})
\end{aligned}$$

Law of Sine

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosine

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos\gamma \\ a^2 &= b^2 + c^2 - 2bc\cos\alpha \\ b^2 &= a^2 + c^2 - 2ac\cos\beta \end{aligned}$$

Angle	0 0	30 $\frac{\pi}{6}$	45 $\frac{\pi}{4}$	60 $\frac{\pi}{3}$	90 $\frac{\pi}{2}$	120 $\frac{2\pi}{3}$	135 $\frac{3\pi}{4}$	150 $\frac{5\pi}{6}$	180 π	210 $\frac{7\pi}{6}$	225 $\frac{5\pi}{4}$	240 $\frac{4\pi}{3}$	270 $\frac{3\pi}{2}$	300 $\frac{5\pi}{3}$	315 $\frac{7\pi}{4}$	330 $\frac{11\pi}{6}$	360 2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1												
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0												
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$												

Definition of the inverse sine function

$$\mathbf{y} = \sin^{-\mathbf{1}}(\mathbf{x}) \Longleftrightarrow \sin(\mathbf{y}) = \mathbf{x}, \text{ where } -1 \le x \le 1, \frac{-\pi}{2} \le y \le \frac{\pi}{2}.$$

Definition of the inverse cosine function $y = \cos^{-1}(x)$

$$\mathbf{y} = \cos^{-\mathbf{1}}(\mathbf{x}) \Longleftrightarrow \cos(\mathbf{y}) = \mathbf{x}, \text{ where } -1 \le x \le 1, 0 \le y \le \pi.$$

Definition of the inverse tangent function $y = \tan^{-1}(x)$

$$\mathbf{y} = \tan^{-\mathbf{1}}(\mathbf{x}) \Longleftrightarrow \tan(\mathbf{y}) = \mathbf{x}, \text{ where } -\infty < x < \infty, \frac{-\pi}{2} < y < \frac{\pi}{2}.$$

Definition of the inverse secant function $y = \sec^{-1}(x)$

$$\mathbf{y} = \sec^{-\mathbf{1}}(\mathbf{x}) \Longleftrightarrow \sec(\mathbf{y}) = \mathbf{x}, \text{ where } (x \le -1) \cup (x \ge 1), y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 0\right].$$