Lecture 13—Sensitivity Analysis Revisited

We left off with an analytic description for the sensitivity of settling velocity with respect to diameter, viscosity, and fluid density:

$$\frac{dw}{w} = \left(\frac{1-m}{2+m}\right)\frac{dD}{D} + \left(\frac{m}{2+m}\right)\frac{d\nu}{\nu} - \left(\frac{\rho_s}{(2+m)}\right)\frac{d\rho}{\rho}$$

So how do diameter and temperature affect settling velocity?

A special case:

What if our sediment particle is smaller than about 0.08 mm and we know that the flow around the particle is completely laminar (in the Stokes range)?

We know exactly the relation between C_D and R:

$$C_D = \frac{24}{R}$$

which means that the slope of the log(C_D) vs log(R) curve is m=-1. Substituting into our general equation, we get:

$$\frac{dw}{w} = 2\frac{dD}{D} - \frac{d\nu}{\nu} - \left(\frac{\rho_s}{\rho_s - \rho}\right)\frac{d\rho}{\rho}$$

so for the Stokes range we see that a 10% variation in particle diameter, *D*, can result in a 20% difference in the estimated fall velocity, *w*.

How about temperature?

Let's find the fall velocity for a 0.4 mm quartz spherical grain falling in water at 20° C, and using our equation determine the fall velocities at 15° C and 25° C. Repeat the analysis for the same sphere falling in air (at sea level).

From the chart for spheres falling in water, we can determine the fall velocity at 20°C:

$$w = 5.85 \text{ cm/s}$$

We construct a look-up table for the fluid and particle properties:

Case 1: Water

	ν (cm^2/s)	ρ (g/cm^3)	ρ_S (g/cm^3)
15°	1.141×10^{-2}	0.9991	2.65
20°	1.005×10^{-2}	0.9982	2.65
25°	0.897×10^{-2}	0.9971	2.65

Calculate R:

$$R(20°C) = \frac{wD}{\nu} = 23.4$$

so from our plot of C_D vs R, m=-2/3

$$\frac{dw}{5.85} = \frac{-2/3}{4/3}\frac{d\nu}{1.005 \times 10^{-2}} - \frac{2.65}{\frac{4}{3}(2.65 - 0.9982)}\frac{d\rho}{.9982}$$

for 15°, then $d\nu = 0.136 \times 10^{-2}$, $d\rho = 8.9 \times 10^{-4}$, and:

$$dw = 5.85\left[\frac{-1}{2}\left(\frac{.136}{1.005}\right) - \frac{3}{4}\left(\frac{2.65}{2.65 - .9982}\right)\left(\frac{8.9 \times 10^{-4}}{.9982}\right)\right] = -.40$$

for 25°, then $d\nu = -0.108 \times 10^{-2}$, $d\rho = -11.7 \times 10^{-4}$, and:

$$dw = 5.85\left[-\frac{1}{2}\left(\frac{-.108}{1.005}\right) - \frac{3}{4}\left(\frac{2.65}{2.65 - .9982}\right)\left(\frac{11.7 \times 10^{-4}}{.9982}\right)\right] = +.32$$

so for 15° it's a –6.8% change

25° it's a +5.5% change

Case 2: Air.

	ν (cm^2/s)	ρ (g/cm^3)	ρs (g/cm^3)
15°	0.145	1.223x10^{-3}	2.65
20°	0.151	1.203x10^{-3}	2.65
25°	0.156	1.183x10^{-3}	2.65

from the chart,

w=305 cm/s

And again, calculate Reynolds number:

$$R(20°C) = \frac{wD}{\nu} = 80.8$$

so from our plot of C_D vs R, m=-1/2

$$\frac{dw}{305} = \frac{-1/2}{3/2}\frac{d\nu}{.151} - \frac{2.65}{\frac{3}{2}\left(2.65 - 1.203\times10^{-3}\right)}\frac{d\rho}{1.203\times10^{-3}}$$

for 15°, then, dw=.66 cm/s

for 25°, then dw=-.01 cm/s

So it doesn't really matter much.

Some conclusions:

- The influence of dν on dw is greater for water than for air
- The influence of dρ is very small in water (0.1%)
- In air, dν and dρ affect dw in opposite directions with similar magnitudes, stabilizing fall velocity
- for a 5° change in temperature:

    --in *water*, dw/w changes about 5%

    --in *air*, dw/w changes about 0.3%

  so temperature effects are about 10 times greater for water than for air.