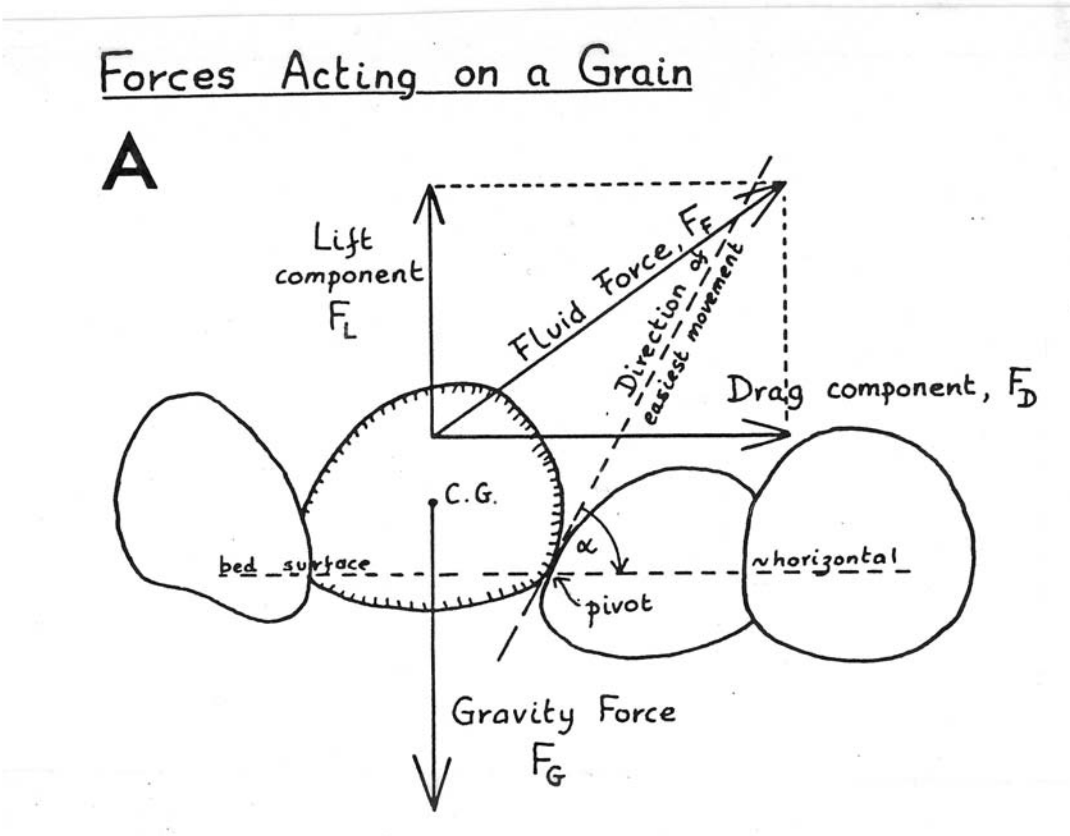
Lecture 17—Initiation of Motion

Let's think about what happens when a grain just starts to move:



There are three forces we're going to worry about:

--Gravity $$F_g = \frac{\pi}{6} D^3 g(\rho_s - \rho)$$

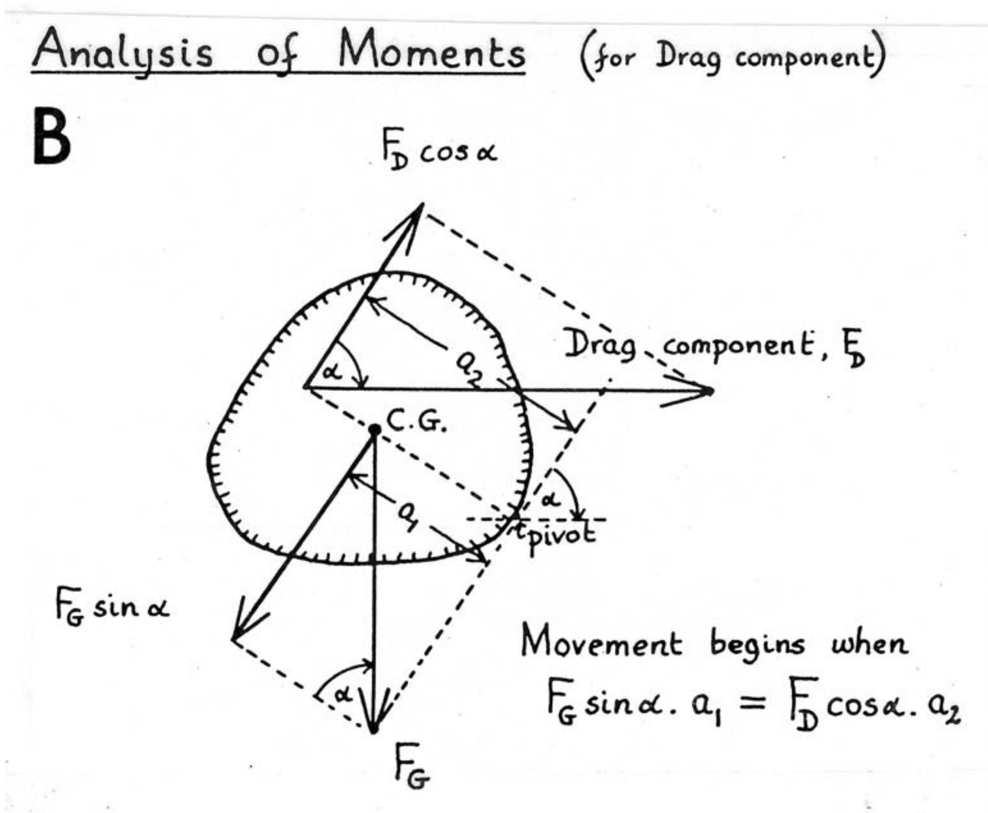
--Drag $$F_d = \frac{1}{2} C_D \rho u^2 A$$

--Lift $$F_l = \frac{1}{2} C_L \rho u^2 A$$

Moreover, there are three ways we could move the particle:

--Lift grain off grains beneath it (requires large lift force; doesn't happen)
--Slide grain along the one downstream from it
--Rotate grain along pivot on downstream grain

Experiments show that the third option is most likely to occur. Unfortunately, when we start talking about forces and rotation, we're going to have to do moments:



(graphics from Middleton and Southard, 1982)

Here we'll just talk about the moment on drag force. We *could* take the trouble to add lift force and drag force to produce a resultant fluid force, then take the component of *that* force acting against the force of gravity during rotation, but since it will have the same *form* as the drag force (albeit with C_L added in) we won't worry too much about it right now (if our answer ends up with C_D in it, we should consider going back and doing this). Motion will begin when the drag force component is *just* balanced by the gravity component, or:

$$a_1\left(F_g \sin\alpha\right) = a_2\left(F_d \cos\alpha\right)$$

Let's take,

$$F_g = c_1 D^3 g\left(\rho_s - \rho\right) \qquad\qquad F_d = c_2 D^2 \tau_0$$

So the equation becomes,

$$a_1 c_1 D^3\left(\rho_s - \rho\right)\sin\alpha = a_2 c_2 D^2 \tau_c \cos\alpha$$

(because motion happens at some critical τ_0, I've changed the subscript to τ_c)

Rearranging,

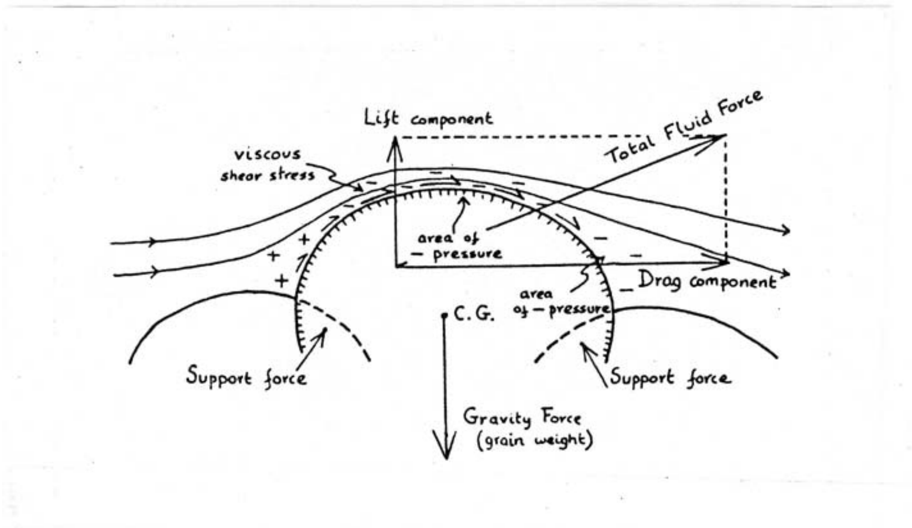
$$\tau_c = \frac{a_1 c_1}{a_2 c_2}\tan\alpha D g\left(\rho_s - \rho\right)$$

And rationalizing,

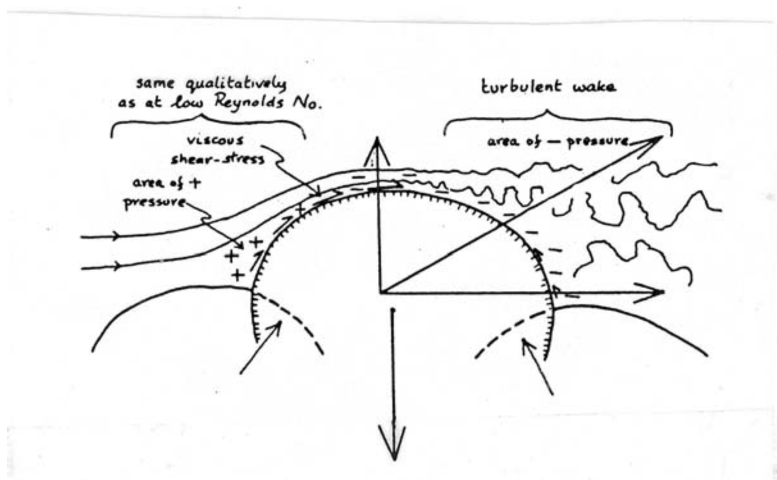
$$\frac{\tau_c}{Dg\left(\rho_s - \rho\right)} = \frac{a_1 c_1}{a_2 c_2}\tan\alpha = \text{some constant}$$

We can call this constant the Shields' parameter, after a famous sedimentologist. Shields' parameter gets a number of different symbols. When it's Shields' parameter, we call it θ or β; when we're just talking about τ_0 (and not τ_c), we often refer to it as nondimensional shear stress, τ_*, and the Shields' parameter as $\tau_{*cr.}$

Curiously, we've just created a dimensionless number that says something about the ratio of drag (and lift) to gravity. This is beginning to look suspiciously like dimensional analysis. All that we'd need to determine criteria for motion is some statement of how the flow acts around the particle of interest. Specifically, does the flow behave laminarly:



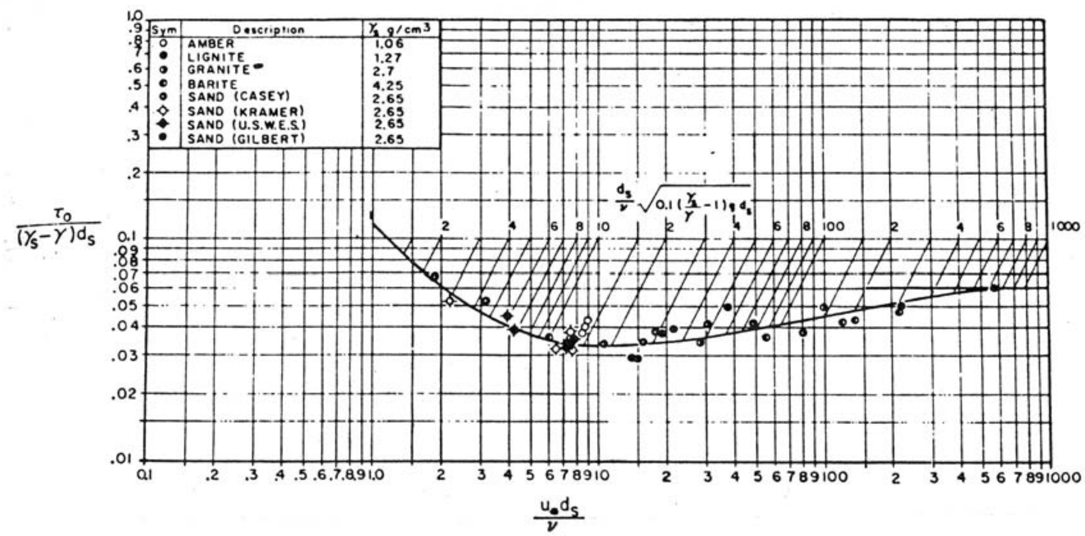
or turbulently:



It turns out that we already *have* such a number. We called it the boundary Reynolds number (R_*) and defined it as:

$$R_* = \frac{u_* D}{\nu}$$

NOW, if we plot R* vs. θ, we can show when motion starts. This diagram is called a Shields' diagram and is our best source for determining what shear stresses are required to produce motion of sediment of a given size and density.



There are several things to note about the Shields' diagram:

--Shear stress (or shear velocity, which is basically the same thing) appears on both axes.

--The data cover a range of R* from 2 to 600. What happens above and below this is not covered by the data (Shields says it becomes slope of –1 below R*=2 and slope of 0 beyond R*=600).

--There's quite a bit of scatter in the data, suggesting that motion happens over some range of shear stresses.

The first point we'll address today, and work on the others during succeeding lectures.

There are two solutions to the problem of having shear stress on both axes. One is to produce a *third* axis that doesn't have shear stress on it. Most Shields' diagrams also have this third axis, added by Vanoni. Another is to recombine R* and θ to produce a third dimensionless variable without shear stress, and plot θ against that (we did this to produce W* and D* for settling velocity). This idea was taken up by Yalin, who did the following:

$$\frac{R_*}{\sqrt{\theta}} = \sqrt{\frac{(\rho_s - \rho)gD^3}{\rho\nu^2}} = \sqrt{\Xi}$$

Where $\sqrt{\Xi}$ is the Yalin parameter. Last little tricky bit. Verify for yourself that the third axis added by Vanoni on your Shields' diagram and the Yalin parameter *are the same thing*. Yep, Vanoni just did a little dimensional analysis and produced a more useful axis as a result.