Lecture 18—Low R_* and High R_* Initiation of Motion

We noted last time that Shields doesn't have data for boundary Reynolds numbers lower than 2. Shields indicated that for $R_*<2$, θ should be inversely proportional to R_*:

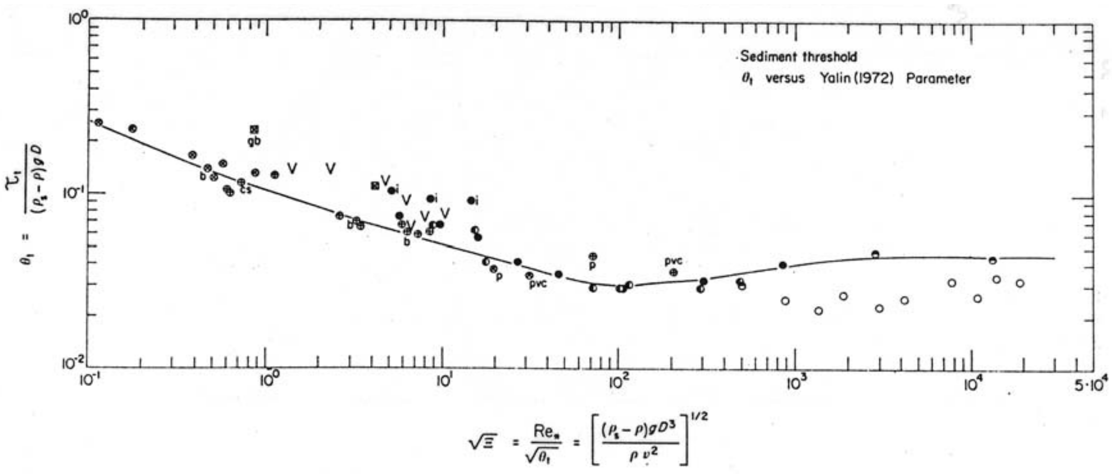
$$\frac{\tau_c}{(\rho_s-\rho)gD}=k_1\frac{\nu}{u_*D}$$

rearranging this yields:

$$\tau_c = k_2{}^3\sqrt{\frac{\mu^2(\rho_s-\rho)^2 g^2}{\rho}}$$

where k_2 is a constant incorporating τ_0 (remember this is constant). The point is that this suggests that τ_c is independent of grain size for grains totally enclosed in the viscous sublayer.

Actual experiments show that this isn't exactly the case. Here's a plot of $\sqrt{\Xi}$ vs. θ for low R_*. If Shields is right, the low R_* part of the graph (on the left) should fall with a slope of –2/3 given these axes. If you look, though, it only falls with a slope of about –1/3, suggesting that θ is still a weak function of grain size. In any case, the density difference becomes more important than grain size for these sizes (for quartz spheres, this is below about 0.167 mm).



What happens for large R_*?

In all truth, there isn't much data for high R_* flows. This is because It's hard to put large objects in a flume, and also because the assumption of a logarithmic velocity profile breaks down for large objects (experimentally, this has been determined to happen at around D/k_s=6 to 10). Without a log profile, it's very hard to estimate u_* in a flume!

For R_*>600 (quartz spheres bigger than about 7mm), u_* is directly proportional to w. In math,

$$u_* = f(w)$$

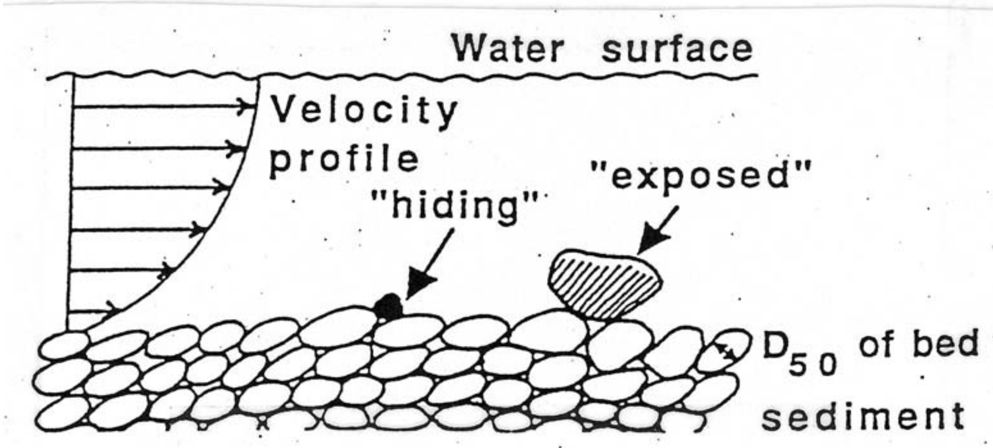
$$\sqrt{\frac{\theta g(\rho_s - \rho)D}{\rho}} = \sqrt{k}\sqrt{\frac{4g(\rho_s - \rho)D}{3\rho C_D}}$$

$$\frac{\theta g(\rho_s - \rho)D}{\rho} = \frac{4kg(\rho_s - \rho)D}{3\rho C_D}$$

$$\theta \propto \frac{1}{C_D}$$

So θ is just a function of C_D! This helps to explain why motion happens over a range of θ; C_D is quite variable as a non-spherical particle rotates.

Unfortunately, when we move to natural rivers this utterly and completely doesn't work. It turns out that people who work in natural rivers need to be concerned with "general" transport on the river, and use D_{50} for the grain size in Shields' criterion. One of the big reasons that the analysis falls down is that the Shields diagram was produced for spheres (or pretty spheric sands) moving on a bed *of similarly sized grains*. If the bed grains are significantly smaller or larger than the moving grain, the shear stress required for motion changes.



Effectively, this changes α:

So, it turns out that we need to know the D_{50} of both the surface and the subsurface grains. Andrews (1983, 1984) gives a relationship for θ based on these:

$$\theta = 0.0834\left(\frac{D_{surface}}{D_{50,sub}}\right)^{-0.872}$$

We can rearrange this using Shields' equation for θ to get τ_{cr}:

$$\tau_{cr} = 0.0834(\rho_s - \rho)g(D_{50,sub})^{0.872} D_{surf}^{\ \ 0.128}$$

Since Andrews' work other studies have produced other values for the exponent of $D_{50,\,sub}$ ranging from 0.9 to 6.0, expressing differences in natural rivers, but all verifying the importance of the grain size of the subsurface layer.

As a result, it's actually easier to move large grains on a bed of smaller grains than to lever smaller grains up and out of the holes created by large bed material.

Well, one more thing. In his original experiments, Shields noted that the bed became rippled almost immediately when movement began. You can imagine that this will affect the local flow regime, and will change the shear stress required for motion. This is because shear stress on the bed comes in two flavors, shear stress from the grains themselves (skin friction shear stress), and shear stress from the bedform (form drag). We'll be discussing the results of this for the next few lectures.