Lecture 22—Introduction to Bed Load

Although most of the sediment carried in a river moved as suspended load, engineers worry quite a bit about bed load. Why is this? Bar formation usually occurs as bed load, for example, which creates a major hazard to navigation. The fate of dredge spoil and the cap of dump sites (New York City still dumps garbage at sea) is also basically a bed load problem. Geologists worry about bed load, too, because many geologic deposits are created by bed load (anything with cross beds or ripple marks, in fact).

There are many ways to approach the study of bed load. In order of their "discovery," if not their current importance:

- "Excess" shear arguments (τ_0-τ_c) or (θ-θ_c) (DuBoys, Shields, MPM)
- Probabilistic arguments (Einstein)
- Stream Power (Bagnold, Yalin)
- Computer Optimization (Brownlie)

We'll look at each in turn, but first, some notation:

g_b = weight flux of sediment / unit width of channel (metric tons/yr/m)

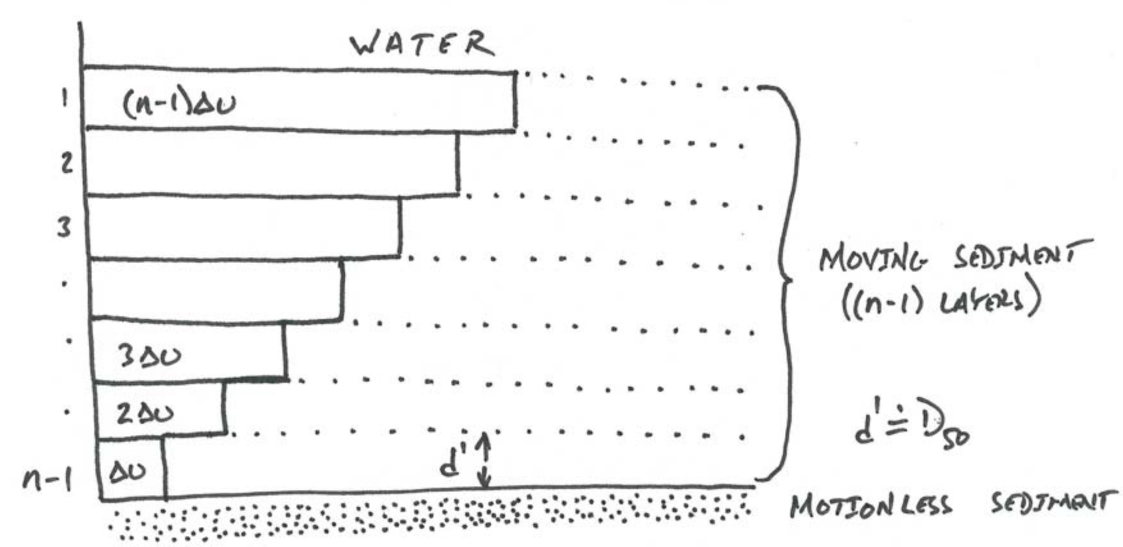
q_b = volume flux of sediment / unit width (cubic meters/yr/m)

ϕ_b = q_* = dimensionless bed load transport rate

$$\phi_b = \frac{q_b}{D^{\frac{3}{2}}\sqrt{\left(\frac{\rho_s}{\rho} - 1\right)g}}$$

--EXCESS SHEAR

DuBoys (1879) visualized the bed as a series of layers:



The bed moves as a series of superimposed layers of thickness *d´* (about the same thickness as D_{50}). The velocity increases linearly (near the bed). If the (n-1)st layer from the top just begins to move at velocity Δu, the (n-2)nd layer (which is moved at Δu with respect to the (n-1)st layer) is moving at 2Δu. Extended up to the surface, the layer moves at (n-1) Δu. The total discharge / unit width = $\bar{u}nd'$.

and, $\bar{u} = \frac{1}{2}(n-1)\Delta u$

or $q_b = \frac{1}{2} nd'(n-1)\Delta u$

and for material of weight *W*,

$g_b = Wq_b$

This doesn't work at the moment, though, because we don't know what *n* is. To get rid of *n*, DuBoys employs the "tractive force method," namely that the shear stress of the flow is equivalent to the friction at the bed:

$$\tau_0 = \rho g R S = f_s(\rho_s - \rho) g n d'$$

where the friction is assumed to be proportional to the submerged weight of the overlying grains. $\tau_0 = \tau_c$ when the top layer just resists motion, essentially when there aren't any moving layers, or when *n*=1. So, $\tau_0 = \tau_c$ when $\tau_0 = f_s(\rho_s - \rho) g d'$, so in general $\tau_0 = n\tau_c$ when (*n*-1) layers are in motion. SO, $n = \frac{\tau_0}{\tau_c}$, which will go into our equation! The result is:

$$g_b = \frac{\Delta u W d'}{2{\tau_c}^2}\tau_0(\tau_0 - \tau_c) = \psi_d \tau_0(\tau_0 - \tau_c)$$

There's a nifty chart for quartz sands that gives values for τ_c and ψ_d.

Many other bed load formulae have basically the same form:

Shields (1936): Data from 2 flumes (40cm and 80cm long), γ_s from 1.06 to 4.2, and D_{50} from 1.56mm to 2.5mm. Shields' data are only for coarse sediment at low shear.

$$g_b = 10\left(\frac{\gamma^2 q S}{(\gamma_s - \gamma)^2 D_{50}}\right)(\tau_0 - \tau_c)$$

Meyer-Peter M∫ller (1948)

The simplest form of Meyer-Peter M∫ller (better known as MPM) is an empirical fit to flume data, but non-dimensionalized:

$$\Phi = q_* = 8(\tau_* - \tau_{*cr})^{\frac{3}{2}}$$

This is a very pretty equation. It's simple, but it shows MPM's heritage as an excess shear type equation. As always, however, it's a little hard to use dimensionless equations, and worse, MPM assumes a flat bed. *However*, MPM has been extensively modified so that it can also incorporate bedforms. We’re going to talk about this next.