Lecture 25—Stream Power and Bed Load

There's an interesting problem with the DuBoys equation (and really, any equation based on excess shear), so interesting that it's actually called the "Einstein paradox"; there's absolutely nothing in DuBoys formulation that prevents the entire bed from going in motion at once. This didn't sit well with some researchers, who reasoned that at least some of the shear stress at the bed gets consumed in intergrain collisions, rather than in picking new grains off the bed. In response, Bagnold split boundary shear stress into two parts:

$$\tau_0 = \tau_{b,s} + \tau_{b,f}$$

where only $\tau_{b,f}$ is responsible for setting grains in motion. If $\tau_{b,f}$ drops below τ_c, (once enough grains are suspended), the bed will be immobile. In other words, the bed load layer acts as "armor" to obtain a stable bed.

Bagnold (and others, later) developed a bed load equation based on the concept of "stream power," that is, the amount of work the stream is capable of performing. Stream power arguments go like this: potential energy is the only source of energy available to move grains, so bed load equations should be based on potential energy. As an example, consider a parcel of water in a stream:

It has potential energy equal to $\rho gz(ydx)$, making the *power* $\frac{\mathrm{d}PE}{\mathrm{d}t} = \frac{\mathrm{d}(\rho gz(y\mathrm{d}x))}{\mathrm{d}t}$. This concept gets used by Bagnold, who says that bed load transport should be related to the stream power per unit bed area, or:

$$\frac{\mathrm{d}PE}{\mathrm{d}t} \times \frac{1}{\mathrm{d}x} = \frac{\mathrm{d}\rho ghzy}{\mathrm{d}t} = \rho gy\frac{\mathrm{dz}}{\mathrm{d}t} = \tau_0 V$$

Yang (1972), on the other hand, uses a different formulation:

$$\frac{\mathrm{d}PE}{\mathrm{d}t} \times \frac{1}{\rho gy\mathrm{d}x} = \frac{\mathrm{dz}}{\mathrm{d}t} = \frac{\mathrm{dz}}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t} = SV$$

So just how does this concept of stream power get worked into a bed load transport equation? Let's use Bagnold's formulation. Consider a set of particles moving over a fixed layer:

$\tau_b = \rho gRS$, which can be decomposed into:

where $\tau_g = (\rho_s - \rho) g V_b \sin\beta$

$\sigma_g = (\rho_s - \rho) g V_b \cos\beta$

Bagnold pits τ_g against a resistive friction force, τ_s:

$$\tau_s = \tan\phi \sigma_g = (\rho_s - \rho) g V_b \cos\beta \tan\phi$$

We know $\tau_b = \tau_{b,s} + \tau_{b,f}$. At the immobile bed interface $\tau_{b,s} >> \tau_{b,f}$ so $\tau_b \cong \tau_{b,s}$.

So, from the force balance above:

$$\tau_{b,s} + \tau_g = \tau_s \qquad \Rightarrow \qquad \tau_{b,s} = \tau_s - \tau_g$$

so,

$$\tau_{b,s} = (\rho_s - \rho) g (\cos\beta \tan\phi - \cos\beta \tan\beta) V_b = (\rho_s - \rho) g \cos\beta (\tan\phi - \tan\beta) V_b$$

Work *required* to move sediment is:

$$W = \tau_{b,s} U_b = (\rho_s - \rho) g \cos\beta (\tan\phi - \tan\beta) V_b U_b$$

whereas the work *available* is:

$$W_a = \tau_0 \overline{u} = \rho g R S \overline{u}$$

and last, the work actually *produced* is:

$$W_p = e W_a$$

so, at long last,

$$q_b = \frac{e \tau_b \overline{u}}{(\rho_s - \rho) g \cos\beta (\tan\phi - \tan\beta)}$$

Yay!

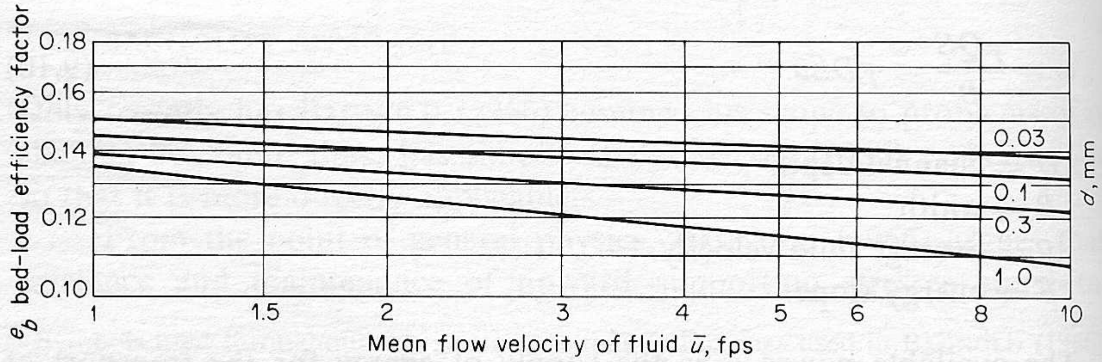

Fig. 9.3 The bedload efficiency factor. [*After* BAGNOLD (*1966*).]