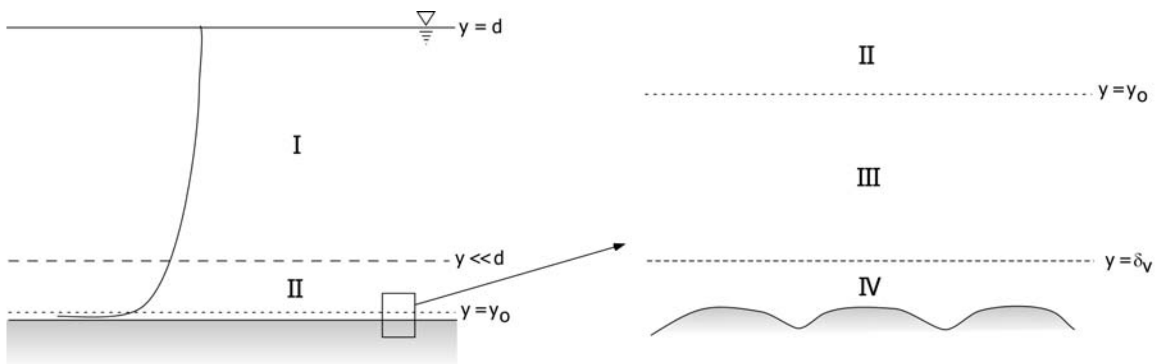
Lecture 6—The Laminar Sublayer

I forgot to deal with a little annoyance we had with the Law of the Wall. We talked briefly about the mystical y_0, some height above the bed for which we *define* the velocity to be 0. We even talked about how large y_0 is, and we called the region below y_0 the laminar sublayer.

Only half true. Let's subdivide the flow into parts:



In the uppermost section (let's call it the *turbulent outer layer*), flow is considered to be vertically constant because of the strong mixing in this zone. This area is 80 to 90% of the flow, and although the law of the wall is specifically not valid here (because of our assumption about τ), the law of the wall is often applied as if it were valid in this region. It's not a *bad* assumption, though, because the law of the wall also shows little velocity change with depth high in the flow.

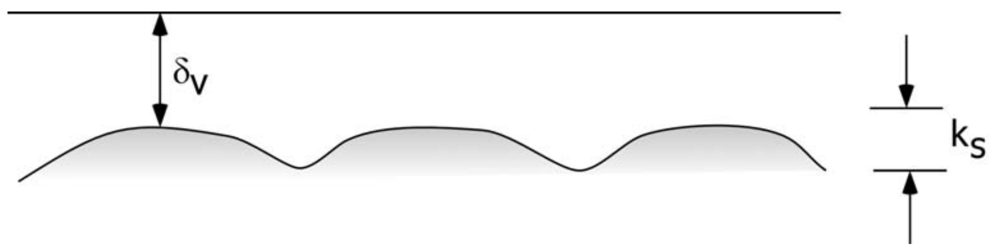
In the next section down (the *turbulent logarithmic layer*), the law of the wall holds. Below that (below y_0), we have a problem. What happens here? In this zone viscosity plays an important role again, and the flow is said to behave in a laminar fashion. Shear stress is constant throughout, leading to an expression for velocity gradient in this lowest layer (the *viscous sublayer* or the *laminar sublayer*):

$$u(y) = \frac{u_*^{\,2}}{\nu} y$$

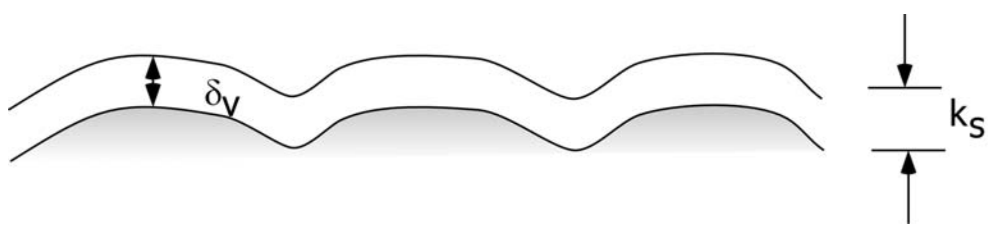
Note that this means that velocity varies linearly in the laminar sublayer! Between the laminar sublayer and the logarithmic layer lies a *transition layer* where both viscosity and turbulence play a role.

Ok, two new things just showed up. First is that I actually had to draw sediment on the bed, something we've managed to avoid so far. The *second* was a mystic symbol, δ_v, marking the top of the laminar sublayer. I managed to avoid a little issue, though—I drew δ_v larger than the height the sediment stuck up off the bed (a height we'll call k_s). There's nothing that says it *has* to be, though. Take two end member ideas:

When $\delta_v > k_s$:



When $\delta_v < k_s$:



The first situation is called *hydraulically smooth* flow, and the other is called hydraulically *rough* flow. Clearly, what's happening at this level will have an effect on sediment transport. So, how thick is δ_v? It's been experimentally determined to be:

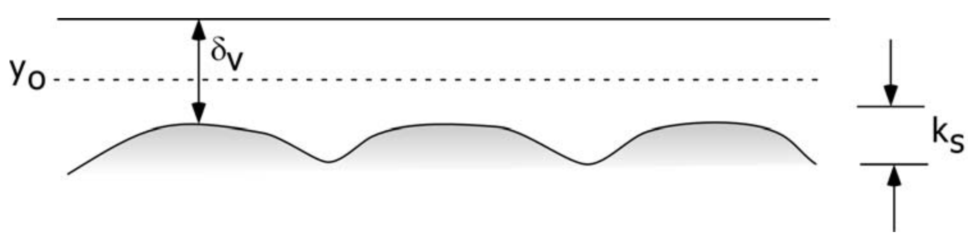
$$\delta_v = 11.6 \frac{\nu}{u_*}$$

So, the more viscous the fluid, the bigger the laminar sublayer is, and the faster the fluid's moving, the smaller it is. This doesn't really tell us the situation, though, because we wanted to know how large δ_v is relative to k_s. We *could* just take the ratio of the two:

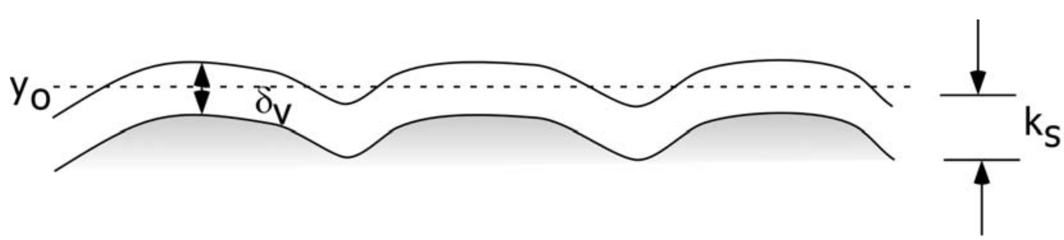
$$\frac{k_s}{\delta_v} = \frac{1}{11.6}\frac{k_s u_*}{\nu}$$

Which has the form of a Reynolds number! When this Reynolds number (we'll call it R_*, the boundary Reynolds number) is large, the flow is rough and the boundary is considered turbulent. When R_* is small, the flow is smooth and the boundary is considered laminar.

Ok, but wait. We already defined y_0, the height above the bed where the velocity is supposed to be 0! Now we're defining a height for the laminar sublayer? What's happening!? It turns out there are two situations that could arise—one is that $y_0<\delta_v$:



and the other is that $y_0>\delta_v$:

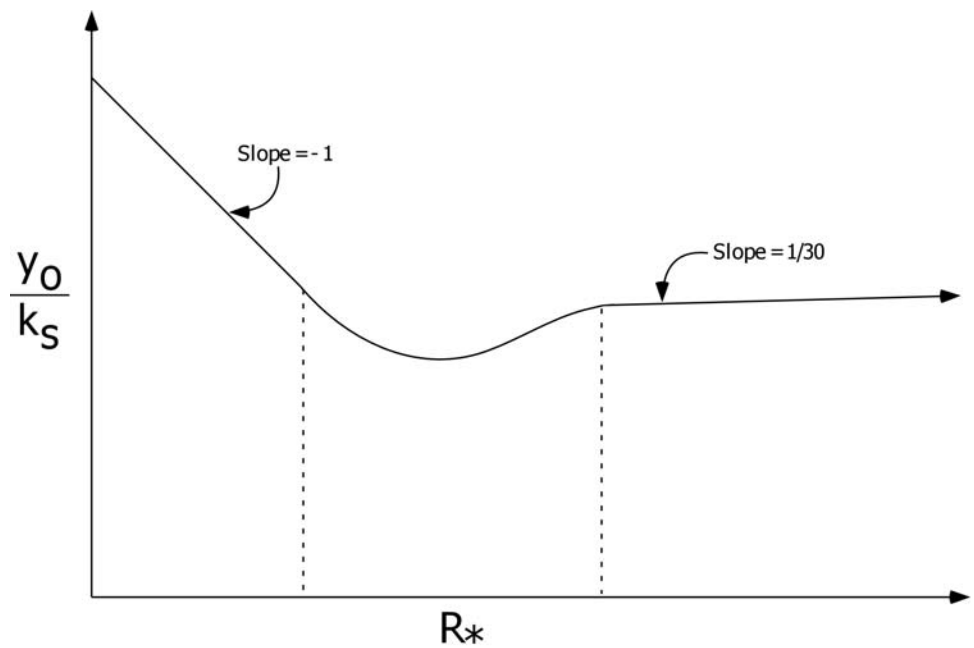


In the first case, the flow is smooth—the roughness elements are more or less contained in the laminar sublayer. In the other, the flow is rough—sediment sticks through the laminar sublayer and y_0 actually gets caught up in the sediment.

A guy named Johann Nikuradse figured out that hydraulically smooth flow happens below R_* of about 5, and above R_* of about 70, the flow is hydraulically rough. He also determined expressions for the height of y_0 in these regions:

$$y_0 = \begin{cases} \dfrac{\nu}{9u_*} & R_* \le 5 \\ \dfrac{k_s}{30} & R_* \ge 70 \\ \dfrac{\nu}{9u_*} + \dfrac{k_s}{30} & 5 < R_* < 70 \end{cases}$$

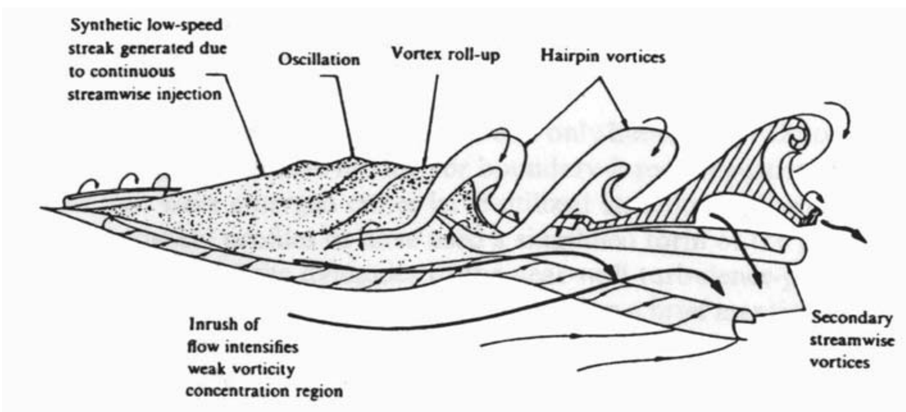
Graphically, it looks sort of like this:



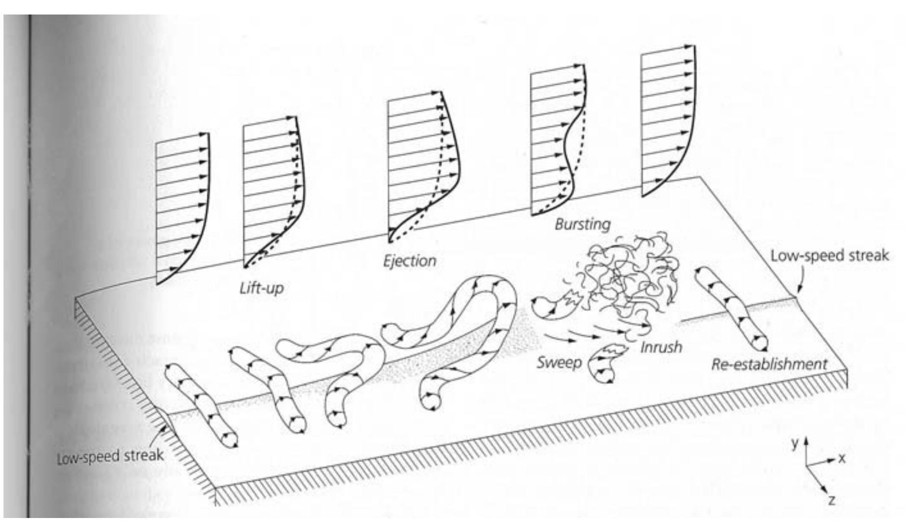
It implies that for low R_*, the "boundary" is effectively the laminar sublayer, and at high R_*, the boundary is the sediment.

We can also now talk about what turbulence is, and where it comes from. We've sort of thrown up our hands until now and said "turbulence is random movement of water packets," but that's not *really* true. It turns out that the interface between the laminar part of the flow and the overlying turbulent part is not smooth. It's sort of wrinkled. Below that interface, water is generally moving slower, and above it, it's moving faster. Fast moving packets of water hit the interface, and deform it—eventually this deformation causes the whole surface to wrinkle.

With time, the wrinkles become more pronounced and less stable. They roll up into a horseshoe of lower speed water ripped up from the laminar layer:

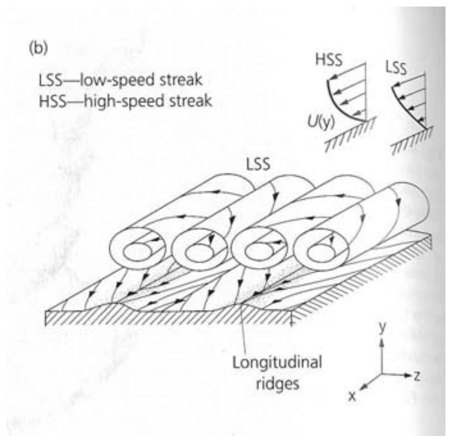


As the horseshoe (or hairpin) vortices lift higher in the flow, the low-velocity packets can't be maintained, and they burst, ejecting low velocity water into the water column.



Presto! Turbulence. Last, bursting is accompanied by *sweeping*, where (once enough low velocity water has been ejected from the boundary layer) high-speed water rushes into the boundary layer. This water loses energy by contact with the bed, becoming low-speed water, and here we go again.

Similarly, researchers have noted that there are *longitudinal vortices* in the boundary layer—where the vortices converge, you get high-speed streaks, and where they diverge, you get low-speed streaks. Convergence also results in (very small scale) extra movement of sand grains, causing very low amplitude sand ridges to form.



These longitudinal vortices are thought to be the "arms" of horseshoe vortices. The spacing of the streaks is governed by (yet another) Reynolds number—in this case a form of boundary Reynolds number with the wavelength of the streaks in place of the thickness of the boundary layer:

$$R_\lambda = \frac{\lambda u_*}{\nu}$$

Where R_λ is constant at about 100, and λ is the spacing between the streaks.

Neat, huh? It turns out that this has bearing on your everyday life. In fact, every time you sit in the cheap seats on an airplane, you can see the effects of turbulence. Next time you got a good fare—look out the window. Chances are good you're sitting on the wing (that's what happens when you get a cheap fare). Now, you'd *think* airplane designers would want the wing to be smooth, to cut down on drag, right? So, what are all those little *fins* all over the wing?

It turns out the little fins are called *vortex generators*, and their whole job in life is to force the boundary layer on the wing to become turbulent. Why would you want to do that? Turbulence increases drag, right?

Only sort of. Yes, the drag on the wing itself increases (this is called the skin friction drag), *but* the size of the “wake” around the wing decreases (this is called the form drag). Form drag is generally a bigger component of drag than is skin friction, so the airplane designers are willing to eat a little more skin friction drag to get less form drag.

There’s another reason, though! Look at the photo again—they didn’t put VGs all over the wing—they only put them in certain spots. Specifically, they put them near the control surfaces of the wing. Why? Turns out the wake causes all sorts of problems—the worst of which is that moving air is kept away from the control surfaces, making them less effective! If the boundary layer can be forced to be turbulent, then fast moving air comes in contact with the wing, and control is maintained. Without VGs, you don’t get a turbulent boundary layer until the airspeed is relatively fast, so slow-speed control is a problem (namely, during takeoff and landing). VGs allow airplanes more control at low speed, and allow for higher angles of attack before stalling.

Oh! Last thing—VGs work well enough on airplanes that now people with too much money are putting them on *cars* for the same reason! Yep. Now you know.