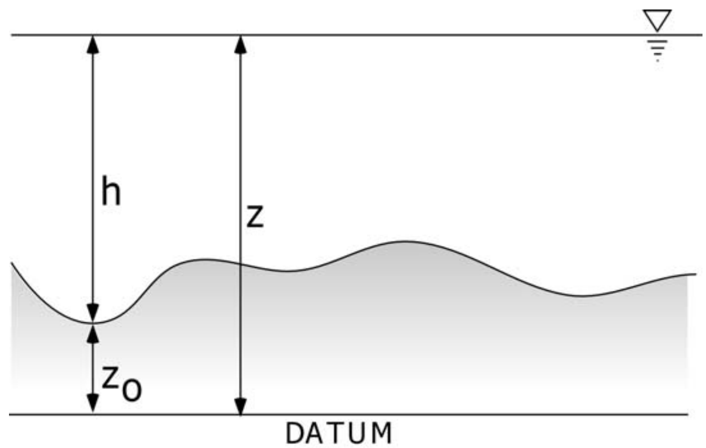
Lecture 8—Specific head.

Return to the Bernoulli equation for open channels;

$$\left(\frac{U^2}{2g}+z\right)_1=\left(\frac{U^2}{2g}+z\right)_2$$

Here, each side is called the *total head*, *H*. However, let's separate *z* into two components:



At this point, we could *define* a component of the total head that only contains the flow depth and the velocity head.

$$H_0=\frac{U^2}{2g}+h$$

This is called the *specific head*. While we're here, I want to redefine specific head in terms of discharge instead of velocity.

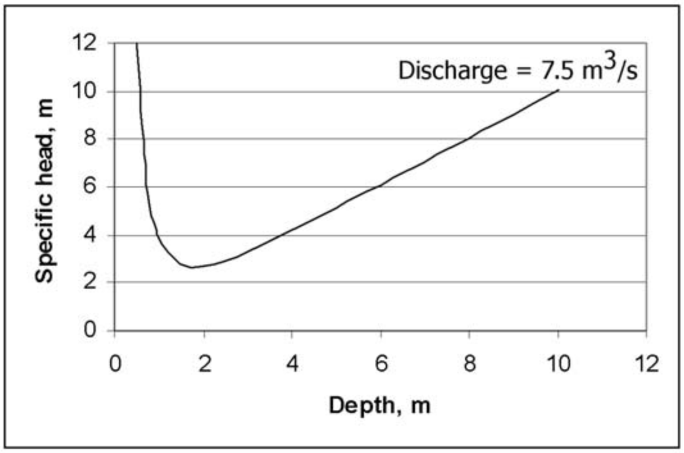
$$H_0=\frac{Q^2}{2gA^2}+h$$

Let's take a simple example of a rectangular channel, here flow doesn't change with width, so:

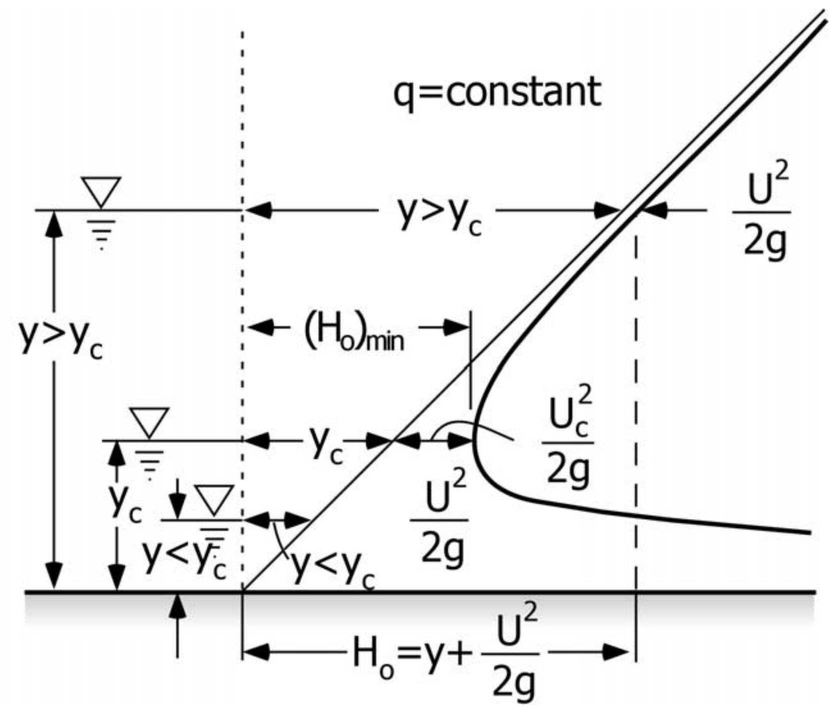
$$H_0=\frac{q^2}{2gh^2}+h$$

(we change from *Q* to *q* to show that it's discharge per unit width)

Unlike total head, specific head doesn't have elevation in it, so evidently it will vary if elevation changes. In this case, since discharge has to remain constant, *flow depth* has to change. Moreover, we can *evaluate* how flow depth changes for any change in H_0. Take a look:



Because this graph has the shape of a parabola (in part), it says that above some critical depth, there are *two* permissible flow depths that will yield an identical discharge. Let's take a look at this where flow depth is actually on the vertical axis like we'd expect:



Notice what this is saying—there's one flow depth where most of the specific head is held as potential energy (h), and just a little is held as kinetic energy ($U^2/2g$), AND there's one where most of the energy is kinetic, and little is potential.

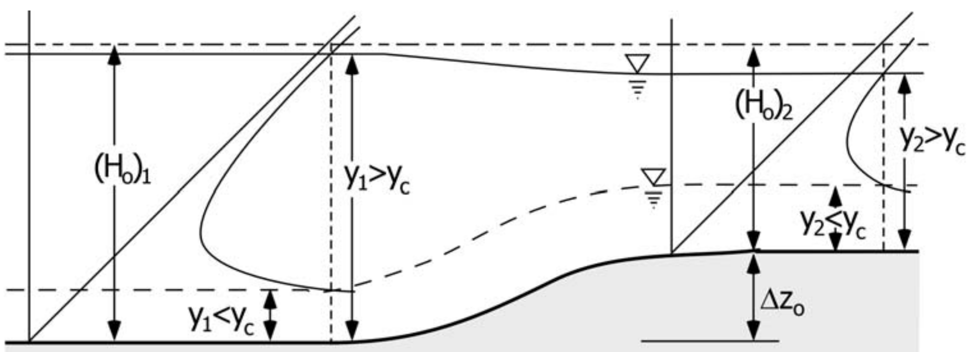
There's also one specific depth and velocity for which energy in the system is minimized. This is the lowest specific head for a given discharge (it says that if the flow is deeper than this, the velocity drops, but if the flow is shallower than this the velocity increases). This point occurs where the derivative of H_0 is 0. So, take the derivative of H_0 with respect to h:

$$\frac{dH_0}{dh} = -\frac{q^2}{gh^3} + 1$$

and substitute $q=uh$ in for discharge

$$1 = \frac{u^2h^2}{gh^3} = \frac{u^2}{gh}$$

Remember that u^2/gh is 1 at the critical depth. Below the critical depth, u^2/gh is more than 1, and above the critical depth, u^2/gh is less than 1. It turns out that u^2/gh is dimensionless, just like Reynolds number! Does it look familiar? This is Froude number (well, actually F^2 in our most common definition of F), and it's clear from this that F=1 separates two important regions of flow.



Above the critical depth (F=1), flow responds to a decrease in specific head by increasing the velocity and decreasing the flow depth. *Below* the critical depth, flow responds to a decrease in specific head by decreasing the velocity and increasing the flow depth. Flow deeper than F=1 is called *subcritical* or *streaming* flow.

Flow shallower than F=1 is called *supercritical* or *shooting* flow. Ok, last bit—what happens if the flow crosses from one region to another? At the transition, the flow doesn't know what to do; it has to suddenly change from one flow depth to the other, resulting (effectively) in a singularity in water depth. Obviously, this can't happen, so the water does the next best thing—it forms a *jump* between one and the other. The two regions are separated by a continuously collapsing wall of water referred to either as a *hydraulic jump*, a *standing wave*, or a *bore*.