When I started the last lecture, I said we had to worry about the fate of rain—some of it ended up evaporating back into the atmosphere, but some of it ended up hitting the ground. What I'd like to talk about today is how we find out how much of it sinks in, and how much of it runs off. This part of the hydrologic cycle provides the critical link between precipitation and runoff, two major themes of hydrology.

So it sort of sucks that we don't have a better linkage. Intuitively, we can talk about what *should* happen, and we can even write an equation that mimics what we said intuitively, but bear in mind that the process our model says should be happening probably doesn't.

So, intuitively, what happens? Mental experiment time—is rain more likely to sink into sand, or into clay? This suggests that some *material property* of the ground affects how fast rain can sink in. The material property in this case is *permeability*, and it's a function not only of how many void spaces there are in a soil, but also how connected they are and how large they are. Typically permeability is determined empirically, with things like constant head tests. What I'm getting at, though, is that if a soil has high permeability, more water will sink in, and low permeability means more runoff.

It gets worse, though. Many of the "connected spaces" are fairly fragile, and one problem is that the impact energy of raindrops can destroy that shallow structure. Also, water that can't sink in sits around on the surface, so *more* water falling from above has no place to go but into the holding area. This decreases the surface area available for infiltration. Another problem is like arteriosclerosis— water actually attaches itself in a thin layer around every grain in the soil, so that the size of the void spaces actually shrinks as the soil gets wetted. Clearly, then, there will be some time dependence on how much water sinks in vs. runs off.

Horton (1940) came up with an equation that satisfies our intuitive notions about how all this should work. Here's the equation:

$$f(t) = f_c + (f_0 - f_c)e^{-kt}$$

where f is the infiltration capacity (in in/hr), f_0 is the initial infiltration capacity, f_c is the final infiltration capacity, and k is an empirical constant that says something about how long it takes for rain to force the soil from its initial to its final infiltration capacity. This has since been experimentally shown to be an effective gauge of infiltration.

Let's have an example, shall we? The initial infiltration capacity of a watershed is estimated as 1.5 in/hr, and the time constant taken to be 0.35 hr^{-1}. The equilibrium capacity is estimated as 0.2 in/hr. What are the values of f at t = 10 min, 30 min, 1 hr, 2 hr, and 6 hr, and what's the total volume of infiltration over the 6 hour time period?

From the Horton equation, we have:

$$f = 0.2 + 1.3\left(e^{-0.35t}\right)$$

Substituting in values of t yields:

t	f
1/6	1.43
1/2	1.29
1	1.12
2	0.85
6	0.36

Which looks like the graph here. The *volume* of water can be found by taking the definite integral under the curve from 0 to 6 hours. Here the integration is easy, and turns out like this:

$$\int_0^6 \left(0.2 + 1.3e^{-0.35t}\right)dt = \left[0.2t + \tfrac{1.3}{-0.35}e^{-0.35t}\right]_0^6$$

plugging and chugging yields an answer of 4.46 inches over the watershed.

All is not sunny in the Horton equation, however. It assumes that rainfall exceeds infiltration rate, so that there *must* be ponding at the surface and reduction in infiltration rate with time. If, however, the

rainfall intensity *doesn't* exceed the rainfall rate, there's no need to drop the infiltration rate. As a result, some researchers have suggested that infiltration capacity should vary with the cumulative infiltration volume and not with time. Unfortunately, this requires iteration between the equation for cumulative infiltration volume (which we got in the example) and the Horton equation. As a result this technique is mostly used in computer simulation.

What if we didn't want to do all this messing around? The dirt simplest way of measuring infiltration is purely empirical. We could simply assume that infiltration is constant during the whole rainfall period, and tune a constant (call it φ) that relates how much water ran off for a given rainfall. The constant would be useable to estimate runoff for future events. Here's an example:

Use the rainfall data listed to determine the φ index for a watershed having a total runoff of 4.9 inches for this storm.

t (hr)	rainfall (in/hr)
0-2	1.4
2-5	2.3
5-7	1.1
7-10	0.7
10-12	0.3

The first step is to make a hyetograph from the data, as shown in the graph. All we need after that is to find the line level that allows the "runoff" part of the hyetograph to be equal to exactly 4.9 inches. In math, the way to do this is:

$$2(1.4 - \phi) + 3(2.3 - \phi) + 2(1.1 - \phi) + 3(0.7 - \phi) + 2(0.3 - \phi) = 4.9$$

You can either solve this equation directly, or go ahead and find φ by trial and error. In this case, assuming φ = 1.5 in/hr yields 2.4 inches of runoff, which is too low; assuming φ = 0.5 in/hr yields 9.0 inches of runoff, which is too high. The answer for this is φ = 1.0 in/hr.

As a final thought, Horton originally wrote that all this surface runoff would take the form of a sheet flow whose depth could be measured—this sheet flow would continue downslope until a stream was encountered.