Lecture 13—An introduction to hydraulics

Hydraulics is the branch of physics that handles the movement of water. In order to understand how sediment moves in a river, we're going to need to understand how *water* moves in a river first. This is easier said than done. In order to start things off, I'd like to start explaining how we approach the movement of fluids. Incidentally, hydraulics, or *fluid mechanics*, is generally a class unto itself, so we're going to be compressing things a bit…

Let's start our analysis of fluids with some pretty basic equations:

--conservation of mass
--conservation of energy

and start by talking about what these mean.

Conservation of mass just says that (barring thermonuclear reactions) you can't create mass or get rid of it. In a river, for example, this means that if so many tons of water enters the beginning of a reach, than so many tons of water *leaves* the reach. Now, we haven't said *how* it leaves. It could leave by evaporation, through seeping into the groundwater, or through animals that come to the bank and drink. *But*, in all likelihood most of the water leaves by flowing out the other end. If we simplify the problem (say by making the river a concrete-lined drainage canal with barbed wire fence around it to keep the animals out, plus it's a cloudy day), then all the water that flows in, flows out. This makes some sense, but how to determine how many tons of water flowed in? Well, we have the cross-sectional area of the river, and we have the *velocity* of the water. Multiplied together, this makes a volume per cubic second. We wanted a mass! To convert from volume to mass, though, you multiply by the density of the fluid. Like this:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

As an example, take water flowing in a canal. At one end, the canal is 10 meters wide, and the flow is 2 meters deep. At the other end, though, the canal is only 5 meters wide and 2 meters deep. The flow velocity at the upstream end is 2 meters per second. What is the velocity at the downstream end? Oh, let's make life interesting—at the upstream end, the water temperature is 20°C, but at the downstream end, the water has chilled to only 5°C. I have no idea why this would happen. {see example}

Notice that although the density of water *does* change with temperature, it doesn't change much (certainly less than air, say). As a result, we often ignore the fluid density, and shorten our equation to:

$$V_1 A_1 = Q = V_2 A_2$$

Notice that now we're talking about a volume per unit time, which is a *discharge*. Still, this is a shorthand form of a basic truth—we can't get rid of mass. There are many other ways to state this, but keep in mind that this is really all we're saying.

Conservation of energy is the other side of the whole conservation of mass thing. Again, barring thermonuclear reactions, you can't create energy or destroy it. You can only shuffle it around into various places. You can store energy as:

--mechanical energy (energy of motion)
--potential energy (energy of position)
--molecular energy (pressure)
--chemical energy
--heat

In fluid mechanics, we won't worry too much about chemical energy, and we'll assume that basically no energy storing reactions are going on in the water. Let's talk about the other ones, though. From a long time ago, you may remember an expression for mechanical (kinetic) energy:

$$KE = \frac{MV^2}{2}$$

and one for potential energy:

$$PE = Mgh$$

where h is the height above some reference elevation.
Pressure is commonly denoted in fluids with just the variable P. It has units of force per unit area.

So, if we ignore heat, we can make an expression for conservation of energy:

$$\frac{M_1 V_1^{\ 2}}{2} + M_1 g h_1 + P_1 = \frac{M_2 V_2^{\ 2}}{2} + M_2 g h_2 + P_2$$

One problem, though; we don't have a convenient means of talking about what M is. We have a fluid that's moving through this system the whole time, so how much water are we talking about? One convenient way around this is to talk about the mass for some given volume, like kilograms per cubic meter, or pounds per cubic foot. This is really the *density* of the fluid, so we can rewrite the equation to be:

$$\frac{\rho_1 V_1^{\ 2}}{2} + \rho_1 g h_1 + P_1 = \frac{\rho_2 V_2^{\ 2}}{2} + \rho_2 g h_2 + P_2$$

Now, we already said that we're going to consider the fluid density to be invariant, so ρ is a constant, and g is a constant. While we're here, let me *define* another constant, γ, where $\gamma = \rho g$. This allows for one *more* rewrite of the equation:

$$\frac{V_1^{\ 2}}{2g} + h_1 + \frac{P_1}{\gamma} = \frac{V_2^{\ 2}}{2g} + h_2 + \frac{P_2}{\gamma}$$

This equation, which is just a statement saying we can't create or destroy energy, is called the *Bernoulli Equation*, and its components are all different places where energy is stored.

Notice that each of these components has units of length. What is the physical meaning of this length? For the pressure term, this makes sense. It's the height to which water would rise in a straw, for example, based on the pressure difference between the water in a cup and the pressure in your mouth. Remember, though, that the water isn't moving! For the *velocity* term, it's the height the water would rise to if it hit a vertical wall—the faster the water is moving, the higher it will rise. The elevation term also makes sense—it's just the change in height from one place to another. Because early water engineers referred to the difference in elevation in wells (which rise to different heights because of differential pressure in the aquifer) as *head*, all of these are called *head*. The first term is called *velocity head*, and it's a statement of the kinetic energy of the system. The second term is called *elevation head*, and it's a statement of the potential energy in the system. The last term is called *pressure head*, and it's a statement of the molecular energy in the system.

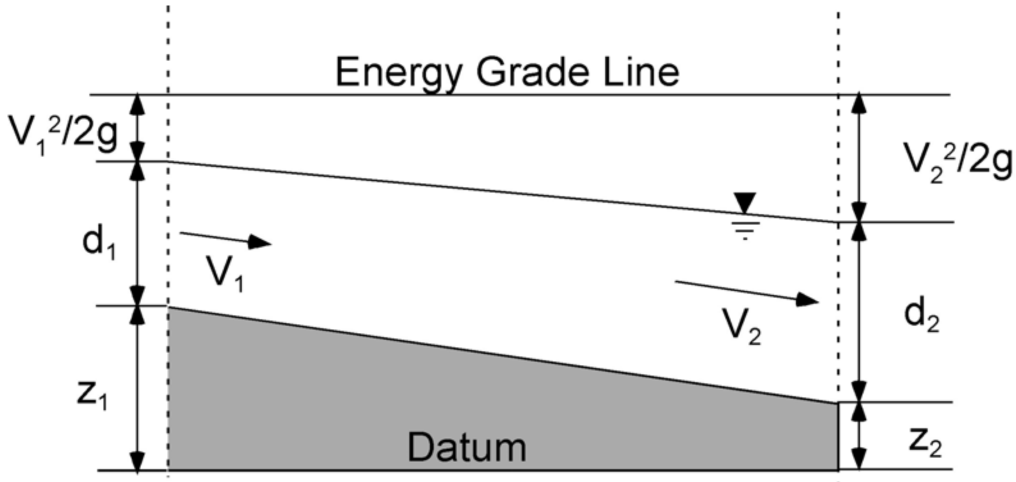
And (of course) an example problem. {see example}.

Ok, now, things get a little easier in what geologists would probably call *unconfined* flows, but which you'll probably hear called *open channel* flows. Basically, water is open to the sky. Because of this, there can be no (meaningful) pressure differences between one section and another, so for *our* purposes, we can be rid of the pressure term!

$$\frac{V_1^{\,2}}{2g} + h_1 = \frac{V_2^{\,2}}{2g} + h_2$$

However, just to be annoying, we often divide the elevation head into two pieces, the elevation of the channel bottom above some datum (maybe sea level, for example) z, and the depth of the water, d. h just equals $z + d$.

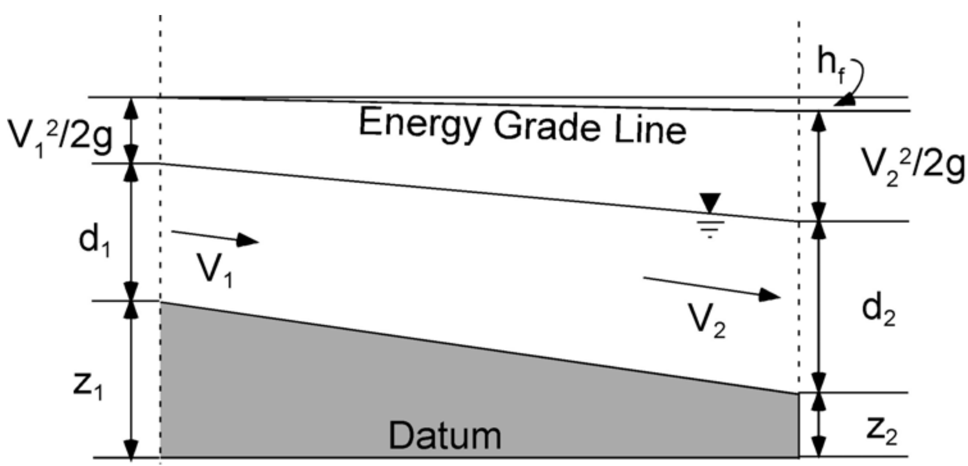
$$\frac{V_1^{\,2}}{2g} + z_1 + d_1 = \frac{V_2^{\,2}}{2g} + z_2 + d_2$$

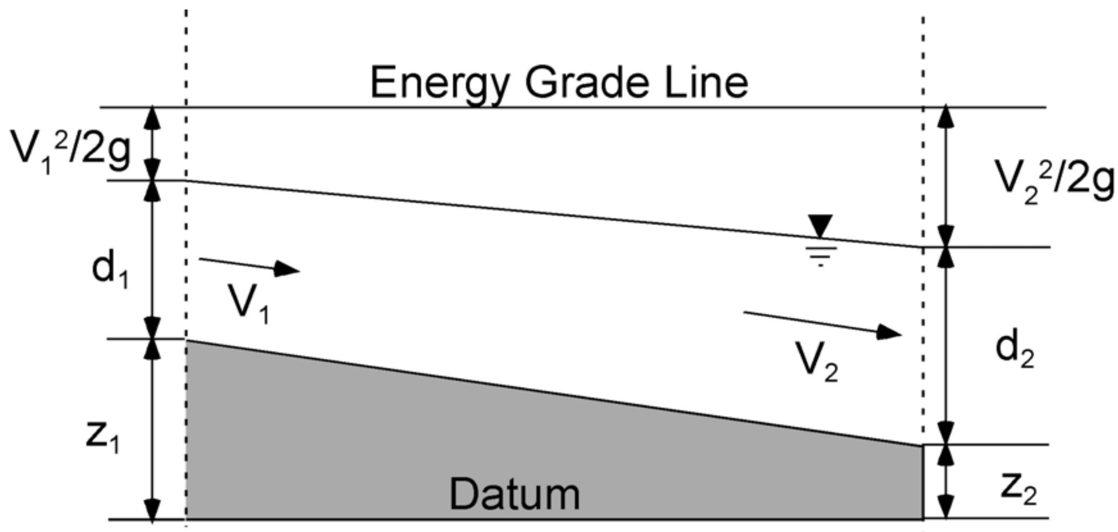


And some terminology; the constant height that all of these things reach (which is a measure of the total energy in the system) is called the energy grade line. Up to now, it has been horizontal, meaning that no energy has left the system. *However*, we haven't dealt with the last form of energy—heat. Throughout this system, energy is being lost as heat because the flowing water comes in contact with the channel sides! This leads to something water engineers call *head loss*. Head loss comes from friction between the fluid and the channel, and it results in the energy grade line having a slight (always negative) slope. So, here's the math:

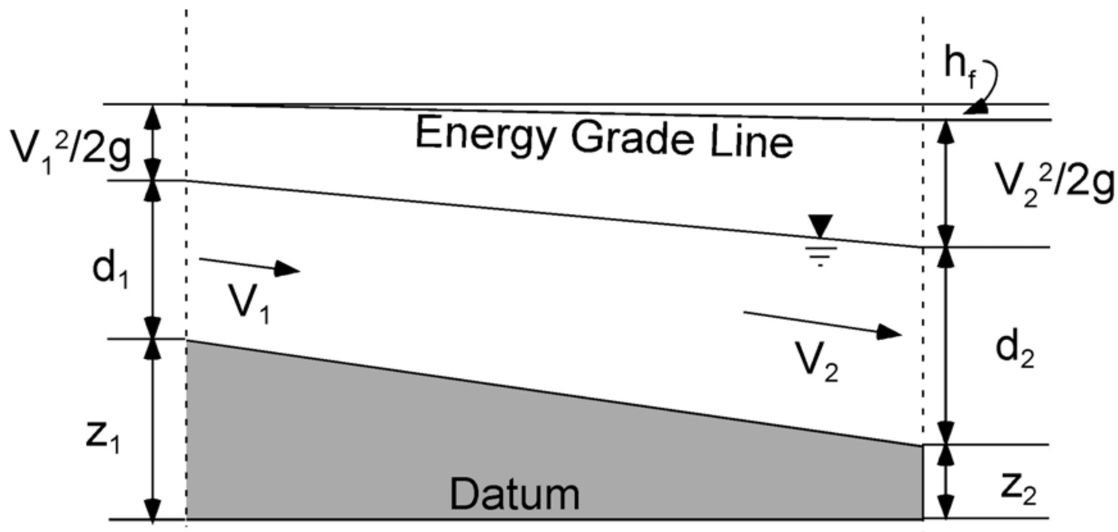
$$\frac{V_1^2}{2g} + z_1 + d_1 = \frac{V_2^2}{2g} + z_2 + d_2 + h_f$$

where h_f is the frictional head loss.





Frictionless open channel Bernoulli diagram



Frictional open channel Bernoulli diagram showing head loss