Lecture 15—Turbulence

We left off last time with the idea that friction isn't the only thing that plays a role in head loss. The other concept was turbulence. So what is turbulence?

Intuitively, let's think about how energy is getting lost from this system. It's not unlike heat being sucked from a coffee cup—heat can be lost directly to the atmosphere by conduction, but this isn't a terribly efficient system. First, you're limited by the amount of fluid in contact with the atmosphere, and second, as the outer part cools, it effectively insulates the inner part from cooling, too. In fact, heat has to diffuse *through* the fluid to get to the place it's going to be conducted from. This is a material property of every fluid.

IF, however, you *stirred* the fluid, all bets would be off. The more you stirred, the more hot fluid would make its way to the top of the coffee, and the more heat would be lost. This is called convection, and it's a lot more efficient than conduction at sucking heat out of coffee.

Why am I telling you all this? It turns out that the exact same processes are at work in our little problem. Instead of heat, however, we're worried about the transfer and loss of *momentum* (remember momentum? that'd be mass times velocity?). Momentum loss by friction is equivalent to conduction—it's slow, and it's based on a material property of the fluid. Here the material property is viscosity, which explains how easy it is to diffuse *momentum* through a fluid. Remember that friction loss only takes place at the fluid boundary with the bed, and in the myriad invisible layers in the fluid. How inefficient is it? An example:

Let's take a fairly standard river:

h = 1.0 m
S = 0.001
μ = 0.001 N·s/m^2

So $\tau_0 = \rho g h S$ = 10 Pa

and $\bar{u} = \frac{\tau_0 h}{3\mu}$ = 3000 m/s

So if friction is the only thing sucking momentum out of a river, a normal river should be flying along at 3000 m/s. Oops.

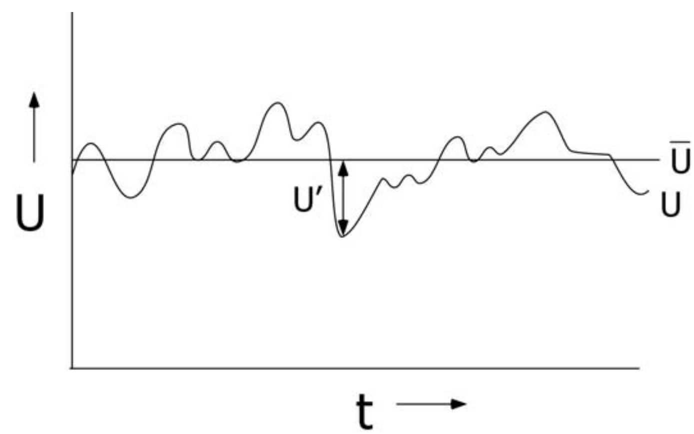
The problem is that a much more effective momentum sucker is at work—it's the equivalent to convection in coffee. High momentum packets of water are being moved up and down in the flow, allowing them to lose momentum (as heat) either in friction, or within the fluid layers themselves.

Soooo, what *is* turbulence?

Turbulence can be expressed as the difference between the instantaneous velocity at some point and a time-averaged velocity (over a suitable interval).

$$u' = u - \overline{u}$$

$$\overline{u'} = \overline{u - \overline{u}} = \overline{u} - \overline{u} = 0$$ ⟵ implies that velocity spends same amount of time above and below the average velocity (this distribution is not necessarily Gaussian)



{drop in the driving analogy here. Tim drives home for the weekend, and it takes him 4 hours to cover the 400 miles, etc.}

Because the time-averaged turbulent excursion tends to be zero, it can be hard to put a number on the size of a turbulent excursion. One solution is to square the excursion first, so that everything is positive, *then* time average, then take the square root of that. This is called the root mean square (RMS) velocity:

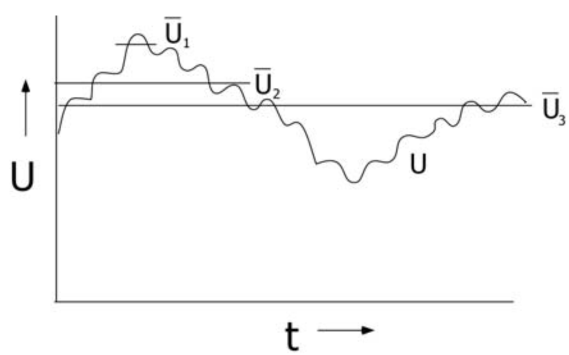
$$\text{RMS}\, u' = \sqrt{\overline{u'^2}}$$

This isn’t as helpful as it could be in part because it’s still dimensional. If the RMS velocity is 2 m/s, is that a lot? It is if the mean velocity is 1 m/s, but it probably isn’t if the mean velocity is 2,000,000 m/s. What we want is a measure of the relative turbulence intensity (RTI); it’s effectively RMS velocity normalized by the mean velocity:

$$\text{RTI} = \frac{\sqrt{\overline{u'^2}}}{\overline{u}}$$

For rivers, $0.01 < \text{RTI} < 0.1$

Choice of averaging time depends on what phenomena we wish to study:



Example (temperature of a classroom)

Point of view

- comfort level of students during lecture
- expansion and contraction of building structure
- winter heating bill

It turns out that turbulence is not completely random. There are spatial correlations in turbulence (large-scale

phenomena like eddies and bursts), and attendant temporal correlations in turbulence.
So how do you tell if turbulence is important to the flow?

In laminar flow, $\tau = \mu \frac{du}{dy}$ so viscous forces are important

In turbulent flow acceleration and large scale motion is important, which is basically inertial force (*F=ma*)

If we wanted to (and we do), we could take the ratio of inertial to viscous forces. When this number is "large," inertial forces are important, and so is turbulence.

This ratio was first described by Osborne Reynolds in the 1880's. We know it as the *Reynolds number*:

$$\mathrm{R} = \frac{\text{Inertial Forces}}{\text{Viscous Forces}}$$

How do we mathematically describe these forces?

--Gravity: $F_g = mg = \rho L^3 g$

--Viscosity: $F_v = \mu \frac{du}{dy} A = \mu VL$

--Inertia: $F_i = ma = \rho L^3 L T^{-2} = \rho V^2 L^2$

So $\mathrm{R} = \frac{\rho \mathrm{V}^2 L^2}{\mu \mathrm{VL}} = \frac{VL}{\mu / \rho} = \frac{VL}{\nu}$

(as an aside, we can also define $\frac{\textit{Inertial Forces}}{\textit{Gravity Forces}}$. This is the *Froude number*. When F>1 waves can't propagate upstream—it's the hydraulic equivalent of a shock wave)

Ok, so now we have a number quantifying how important turbulence is in the system. We still don't know how large R has to be before the system is turbulent! Truthfully, that's on purpose. We’ve left the definition of R vague—we still don’t know what *L* is, for example. That’s because there are several different Reynolds numbers, depending on what you define *L* to be. In our case, though, *L* will be equal to the flow depth, and *V* to the flow velocity.