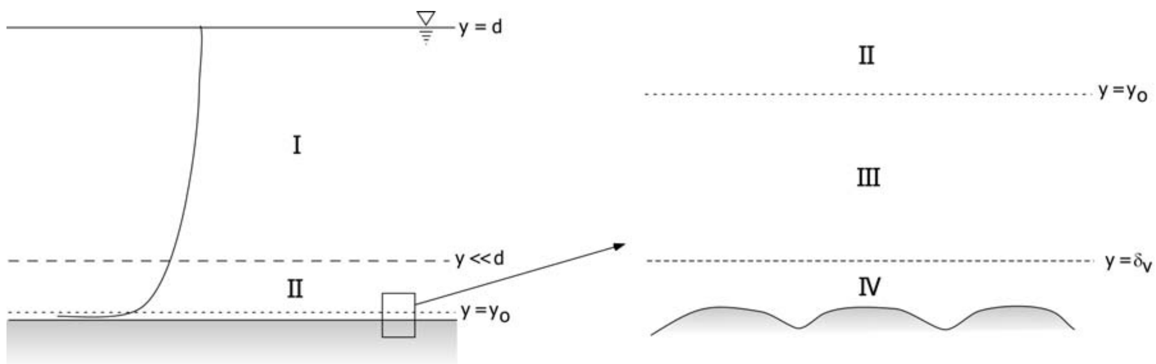
Lecture 16a—The Laminar Sublayer

I forgot to deal with a little annoyance we had with the Law of the Wall. We talked briefly about the mystical y_0, some height above the bed for which we *define* the velocity to be 0. We even talked about how large y_0 is, and we called the region below y_0 the laminar sublayer.

Only half true. Let's subdivide the flow into parts:



In the uppermost section (let's call it the *turbulent outer layer*), flow is considered to be vertically constant because of the strong mixing in this zone. This area is 80 to 90% of the flow, and although the law of the wall is specifically not valid here (because of our assumption about τ), the law of the wall is often applied as if it were valid in this region. It's not a *bad* assumption, though, because the law of the wall also shows little velocity change with depth high in the flow.

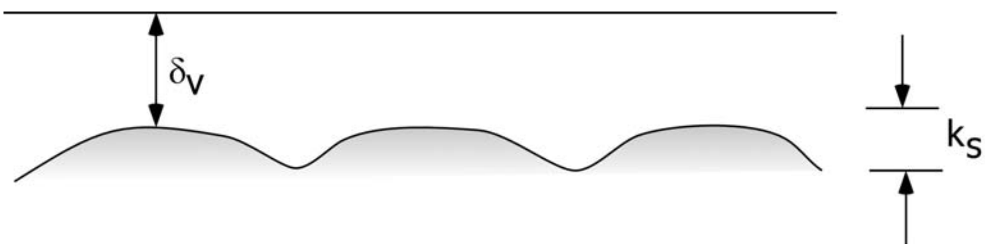
In the next section down (the *turbulent logarithmic layer*), the law of the wall holds. Below that (below y_0), we have a problem. What happens here? In this zone viscosity plays an important role again, and the flow is said to behave in a laminar fashion. Shear stress is constant throughout, leading to an expression for velocity gradient in this lowest layer (the *viscous sublayer* or the *laminar sublayer*):

$$u(y) = \frac{{u_*}^2}{\nu} y$$

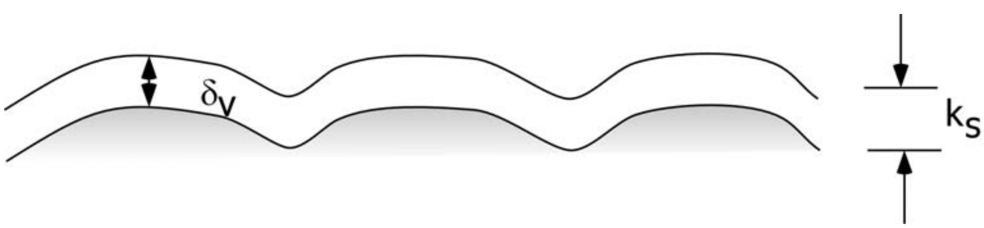
Note that this means that velocity varies linearly in the laminar sublayer! Between the laminar sublayer and the logarithmic layer lies a *transition layer* where both viscosity and turbulence play a role.

Ok, two new things just showed up. First is that I actually had to draw sediment on the bed, something we've managed to avoid so far. The *second* was a mystic symbol, δ_v, marking the top of the laminar sublayer. I managed to avoid a little issue, though—I drew δ_v larger than the height the sediment stuck up off the bed (a height we'll call k_s). There's nothing that says it *has* to be, though. Take two end member ideas:

When $\delta_v > k_s$:



When $\delta_v < k_s$:



The first situation is called *hydraulically smooth* flow, and the other is called hydraulically *rough* flow. Clearly, what's happening at this level will have an effect on sediment transport. So, how thick is δ_v? It's been experimentally determined to be:

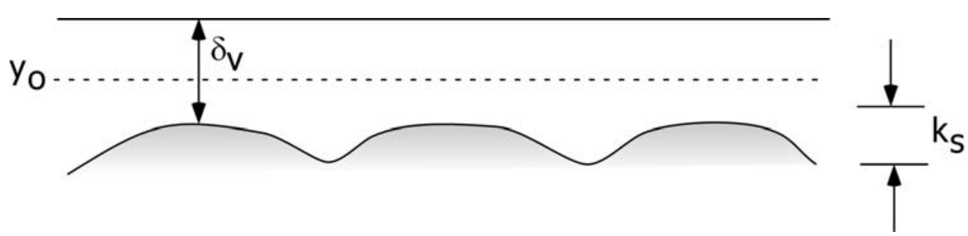
$$\delta_v = 11.6\frac{\nu}{u_*}$$

So, the more viscous the fluid, the bigger the laminar sublayer is, and the faster the fluid's moving, the smaller it is. This doesn't really tell us the situation, though, because we wanted to know how large δ_v is relative to k_s. We *could* just take the ratio of the two:

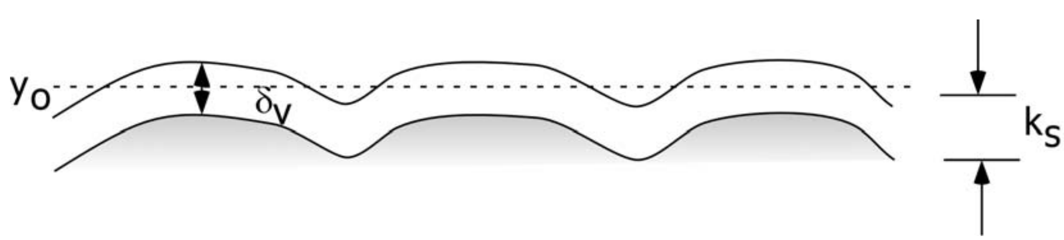
$$\frac{k_s}{\delta_v} = \frac{1}{11.6}\frac{k_s u_*}{\nu}$$

Which has the form of a Reynolds number! When this Reynolds number (we'll call it R_*, the boundary Reynolds number) is large, the flow is rough and the boundary is considered turbulent. When R_* is small, the flow is smooth and the boundary is considered laminar.

Ok, but wait. We already defined y_0, the height above the bed where the velocity is supposed to be 0! Now we're defining a height for the laminar sublayer? What's happening!? It turns out there are two situations that could arise—one is that $y_0 < \delta_v$:



and the other is that $y_0 > \delta_v$:

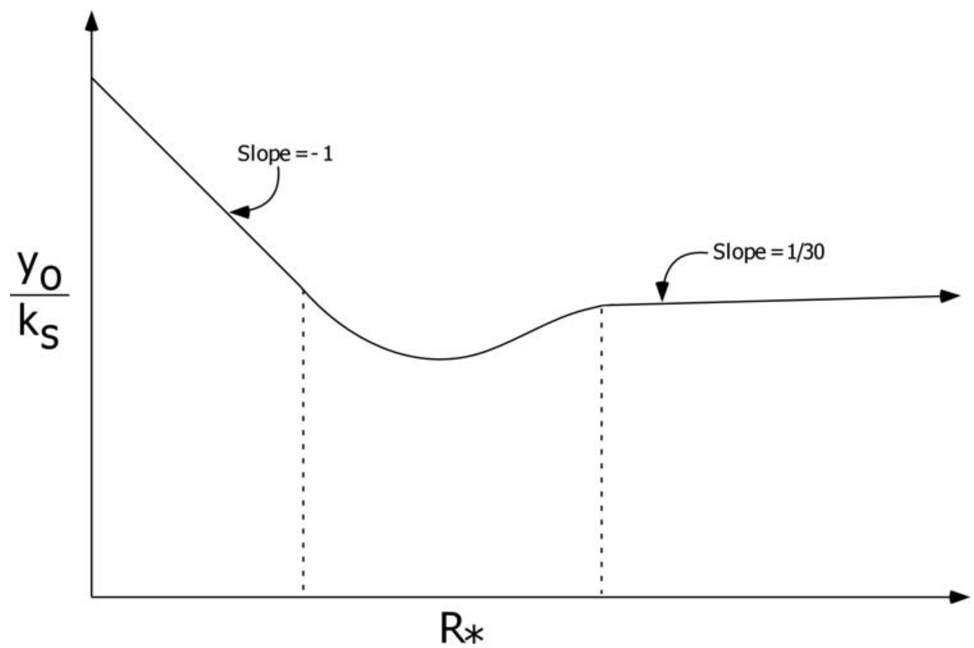


In the first case, the flow is smooth—the roughness elements are more or less contained in the laminar sublayer. In the other, the flow is rough—sediment sticks through the laminar sublayer and y_0 actually gets caught up in the sediment.

A guy named Johann Nikuradse figured out that hydraulically smooth flow happens below R_* of about 5, and above R_* of about 70, the flow is hydraulically rough. He also determined expressions for the height of y_0 in these regions:

$$y_0 = \begin{cases} \dfrac{\nu}{9u_*} & R_* \le 5 \\ \dfrac{k_s}{30} & R_* \ge 70 \\ \dfrac{\nu}{9u_*} + \dfrac{k_s}{30} & 5 < R_* < 70 \end{cases}$$

Graphically, it looks sort of like this:



It implies that for low R_*, the “boundary” is effectively the laminar sublayer, and at high R_*, the boundary is the sediment.