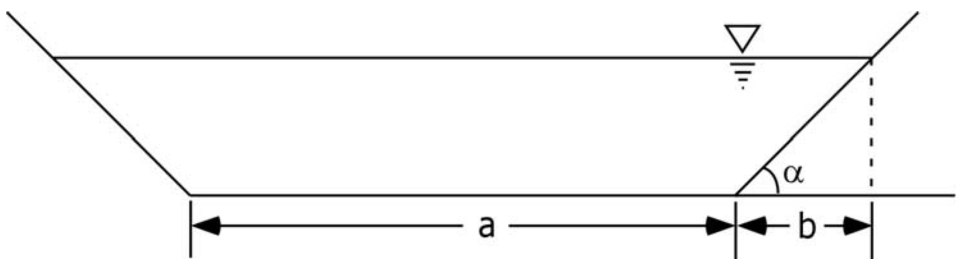
Lecture 18—Stage, discharge, and the hydrograph

We left off with an expression relating velocity to measurable parameters like slope and radius, and one empirical constant—Manning's n. We called this equation the Chézy-Manning equation:

$$U = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

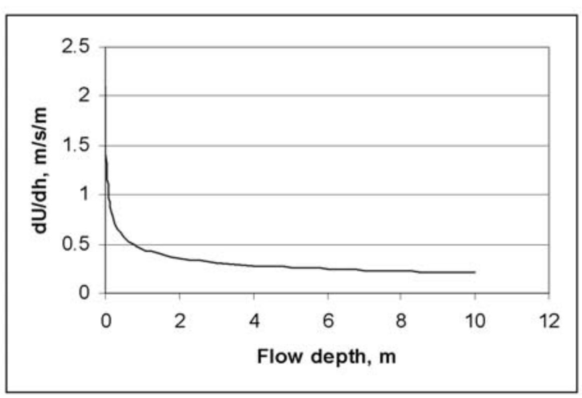
Let's ask a strange question. What happens to velocity as the depth of the river increases? For simplicity's sake, we'll call our river channel a wide trapezoid, such that $A = (a+b)h$ and assume that $R \approx h$.



In this case, an increase in depth results in some strange increase in velocity, namely:

$$\frac{dU}{dh} = \frac{2}{3n} h^{\frac{-1}{3}} S^{\frac{1}{2}}$$

What's this curve look like?



Basically, this says that each marginal increase in water depth has less of an effect on the water velocity.

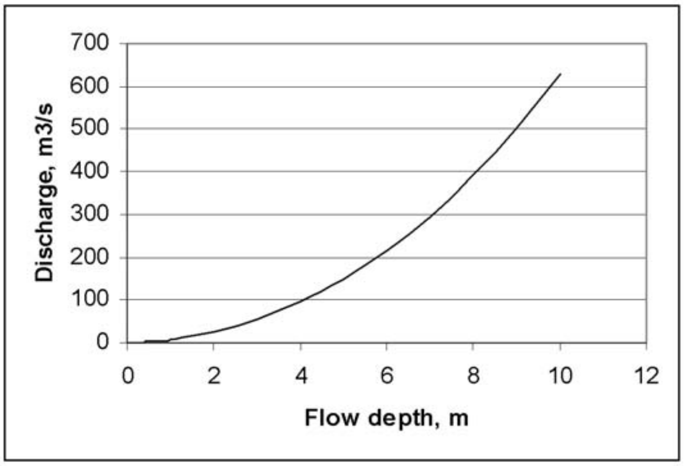
What would be the discharge for this flow? Remember that,

$$Q = UA$$

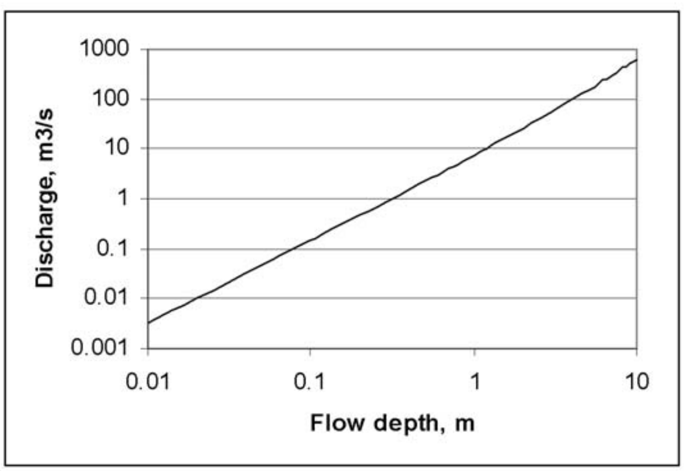
So,

$$Q = \frac{1}{n} h^{\frac{2}{3}} S^{\frac{1}{2}} (a+b)h = \frac{1}{n} h^{\frac{5}{3}} S^{\frac{1}{2}} (a + h\cot\alpha)$$

What's this look like?



This shows that as height increases, discharge increases more. It's almost exponential, but not really. In fact, take a look at what happens if you plot it on a log-log plot.



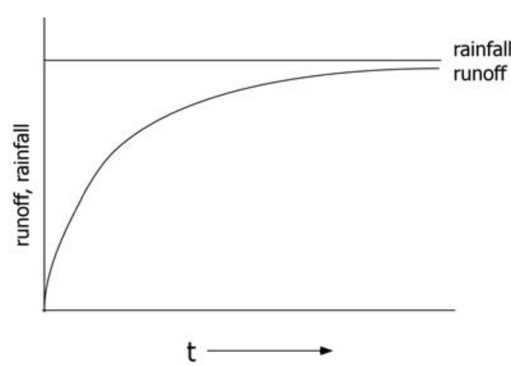
This is a fairly important graph. It shows what happens to discharge when depth increases, which is nice because *depth* is fairly easy to measure, sorta. We *could* install a gauge that measures nothing but water depth in the gauge, and report a height in *units above the bottom of the gauge*. This is called *stage*, and a graph like this is called a *stage-discharge* graph.

In our example everything worked ideally, because it was based on a nice mathematical approximation for a channel. In real life, however, the channel has some topography of its own, and when you overtop a bar, for example, the relationship between depth and area won't be simple. *Worse*, that bar probably had vegetation on it, so the channel roughness also changes! This affects *n*, so the relationship can become rather strange. As a result, it is common for a stream to be gauged and calibrated for many years, and a rating curve developed empirically, but using the same basic form as the one we developed mathematically.

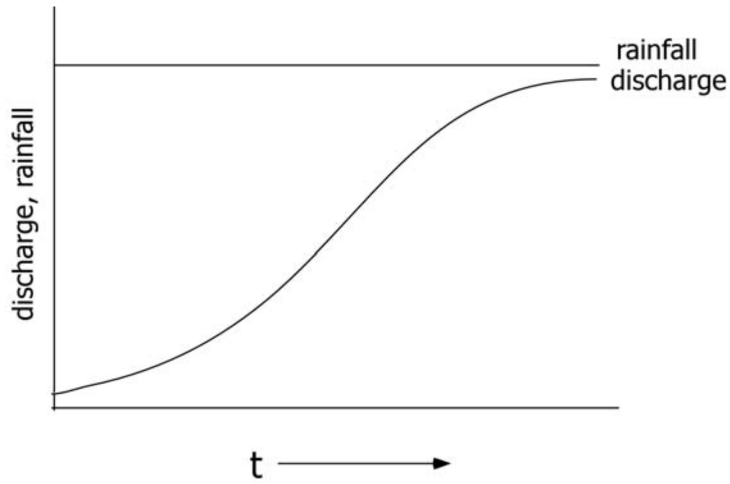
One thing worth mentioning—what happens to our curve when the flow exceeds the limits of our trapezoidal channel? The depth at which the channel is completely full is called the "bankfull depth", and it occurs, naturally, at bankfull discharge. Once this discharge is exceeded, we'll have a flood. More on this later.

So, we have a curve that relates depth of water to discharge. But wait a second—we *also* have a curve that talks about how to relate amount of precipitation to depth of water! If we *joined* these two together, things might get very interesting. Let's have a think…

From before, we had the idea that if rain goes on long enough, eventually all water entering the watershed exits via the stream at the same rate:



We now also have a diagram that says for the same amount of rain, we get some constant discharge. We could combine these to make a diagram relating rainfall to discharge in the river:



The net result is something vital to this class—the hydrograph.