Lecture 22—Synthetic Hydrographs

Ok, we were getting all het up last time, what with the *convolution* and the *deconvolution*, and we were actually in the business of creating storm hydrographs (and therefore *predicting* discharge) from hyetographs; at least when we had a unit hydrograph to work from. Now, having a unit hydrograph implies that you have at least *some* gauging—but most of the rivers in the U.S., leave *aside* the rivers of the world, have no gauges on them. How to get a UH for these rivers?

Simple. We *synthesize* one from beliefs about how rain infiltrates. Yep. Now, obviously this is not simple, and equally obviously, there are a number of different ways of doing it, all based on assumptions about how watersheds work. We'll take a look at some of these, and talk about where they come from, but bear in mind that there is no one way to do this, and that every region does it differently (and probably empirically). Basically, all these methods involve making some statements about basic hydrograph parameters (lag time, time base, etc.) and fitting a curve to that.

One way of doing this is to make some assumptions about t_p and T_b, and then start sketching. Snyder thought this one up in 1938, based on a study of watersheds in the Appalachian Highlands. Here's a set of relationships:

$$t_p = C_t \left(LL_c\right)^{0.3}$$

where L is the length of the main channel from outlet to divide (in miles), L_c is the length of the main stream from outlet to a point nearest the centroid of the watershed, and C_t is a coefficient ranging from 1.8 to 2.2 (in the Appalachian Highlands—it has been found to vary from 0.4 in mountainous areas to 8.0 along the Gulf of Mexico). C_t says how quickly runoff is transmitted to the channel of the watershed.

$$Q_p = \frac{640 C_p A}{t_p}$$

where Q_p is the peak discharge, A is the watershed area, and C_p is a storage coefficient ranging from 0.4 to 0.8, which larger values of C_p associated with smaller values of C_t. C_p says something about

how much storage is available in the system, but larger values mean less storage.

$$T_b = 3 + \frac{t_p}{8}$$

where T_b is the time base of the hydrograph, in days.

Last up, to know what *kind* of UH you just made, use:

$$D = \frac{t_p}{5.5}$$

To get the duration of the rainfall. You can then adjust to the UH you wanted either by using the S-curve method, or by using the adjusted equation:

$$t'_p = t_p + 0.25(D' - D)$$

where *D'* is the hydrograph duration you wanted…

Wait! two more curves just to make life easier.

$$W_{75} = 440\left(\frac{Q_p}{A}\right)^{-1.08}$$

$$W_{50} = 770\left(\frac{Q_p}{A}\right)^{-1.08}$$

where *Wn* is the hydrograph width at n% of the peak discharge, and the width is oriented so that 1/3 of the width is in front of the peak, and 2/3 after the peak. That's it! From here, you sketch the hydrograph, being sure to keep the area under the curve equal to 1 inch of rainfall.

Let's do an example—given an Appalachian watershed with an area of 100 mi^2, develop the synthetic 2-hour unit hydrograph. Here's the salient lengths and coefficients:

C_t = 1.08   L = 18 miles
C_p = 0.6 L_c = 10 miles

{run through this}

Ok, the one issue with this method is how on earth to keep the area right and sketch the curve. The Soil Conservation Service (SCS) came up with a solution—just draw it as a triangle. {see chart} This idea tends to work well for small watersheds. From the definition sketch:

$$R = \frac{Q_p T_R}{2} + \frac{Q_p B}{2}$$

where R is the rainfall excess in inches, T_R is the time of rise, and $B = T_B - T_R$. Rearranging,

$$Q_p = \frac{2R}{T_R + B}$$

now, some empiricism. From "a large number of hydrographs,"

$$B \approx 1.67 T_R$$

So,

$$Q_p = \frac{0.75R}{T_R}$$

Conversion from inches of runoff to cfs of discharge requires the watershed area and the factor (1 in/hr = 645 cfs/sq mi), resulting in:

$$Q_p = \frac{484 A R}{T_R}$$

Last bit! We already know that:

$$T_R = \frac{D}{2} + t_p$$

So,

$$Q_p = \frac{484 A R}{\dfrac{D}{2} + t_p}$$

And we're left wondering only what *tp* is. Here life gets ugly in SCS land. A *highly* empirical formula….

$$t_p = \frac{\ell^{0.8}(S+1)}{1900\,y^{0.5}}$$

where ℓ is the distance along channel from outlet to the divide (in feet), *y* the average watershed slope in %, and *S* the *potential abstraction* and equal to $S=(1000/CN)-10$, where *CN* is the curve number as shown.

Land Use Description	Hydrologic Soil Group			
	A	B	C	D
Cultivated Land w/o conservation treatment	72	81	88	91
Cultivated Land w/ conservation treatment	62	71	78	81
Pasture or range land in poor condition	68	79	86	89
Pasture or range land in good condition	39	61	74	80
Meadow (good condition)	30	58	71	78
Thin stand of forest or wood, poor cover	45	66	77	83
Forest or wood, good cover	25	55	70	77
Open spaces (lawns, parks, etc.) good condition, grass on 75% or more of area	39	61	74	80
Open spaces (lawns, parks, etc.) fair condition	49	69	79	84
Commercial and business areas (85% impervious)	89	92	94	95
Industrial districts (72% impervious)	81	88	91	93
Residential (1/8 ac lots, 65% imperv.)	77	85	90	92
(1/4 ac lots, 38% imperv.)	61	75	83	87
(1/3 ac lots, 30% imperv.)	57	72	81	86
(1/2 ac lots, 25% imperv.)	54	70	80	85
(1 ac lots, 20% imperv.)	51	68	79	84
Paved parking lots, driveways, etc.	98	98	98	98
Paved roads w/ storm sewers	98	98	98	98
Gravel roads	76	85	89	91
Dirt roads	72	82	87	89

Where A, B, C, and D refer to "sandy well drained soils," "sandy loam," "clay loam or shallow sandy loam," and "poorly drained heavy plastic clay," respectively.

This should leave you somewhat cold. I mean, we *know* hydrographs don't look like triangles! One solution to this comes (again) from the SCS—they took a lot of hydrographs, and averaged them. The result is a hydrograph whose *shape* is governed by the average of all these hydrographs the SCS has, but whose *peak* and *lag time* are determined by you (either by Snyder's, SCS, or some other method). Here's the SCS diagram:
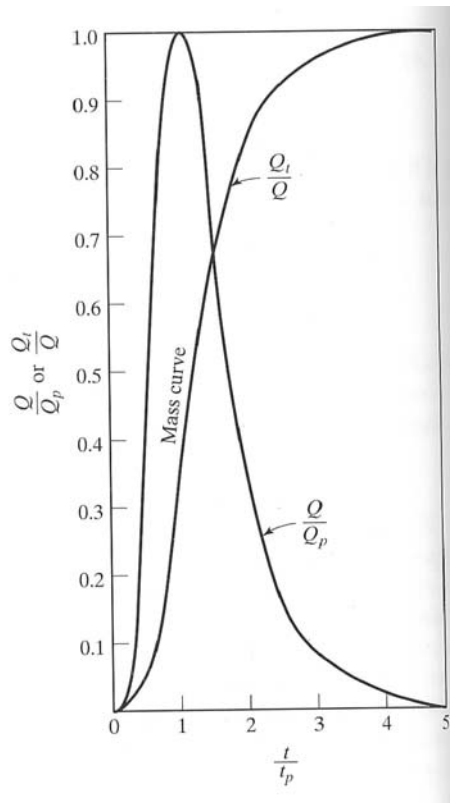
There's also a chart:

t/t$_p$	Q/Q$_p$	t/t$_p$	Q/Q$_p$
0	0	1.4	0.75
0.1	0.015	1.5	0.66
0.2	0.075	1.6	0.56
0.3	0.16	1.8	0.42
0.4	0.28	2.0	0.32
0.5	0.43	2.2	0.24
0.6	0.60	2.4	0.18
0.7	0.77	2.6	0.13
0.8	0.89	2.8	0.098

0.9	0.97	3.0	0.075
1.0	1.00	3.5	0.036
1.1	0.98	4.0	0.018
1.2	0.92	4.5	0.009
1.3	0.84	5.0	0.004

There's also another way, but you're not going to like it. Some hydrologists noted the similarity between a specific kind of statistical distribution and the shape of a hydrograph. The distribution is called a *gamma* distribution, and it's defined as:

$$f(x) = \frac{x^{\alpha-1} e^{\frac{-x}{\beta}}}{\beta^{\alpha} \Gamma(\alpha)}$$

Ok, I know, it's not pleasant to look at. If you substitute t for x, though, and say that the output is supposed to be Q, then the miracle of the gamma distribution is that t_p is given by $\alpha\beta$. With a bunch of substitutions and a little calculus, then, you can come up with:

$$Q_p = \frac{C_v A \alpha^{(\alpha+1)}}{t_p e^{\alpha} \Gamma(\alpha+1)}$$

Where A is the watershed area, and C_v is a constant equal to 1.008. SO, we can just gather up the whole gamma distribution part of that mess and put it to one side:

$$Q_p = \frac{C_v A}{t_p} \phi(\alpha)$$

The good news from *that* is that you could rearrange this to put the stuff you know (Q_p, C_v, A, and t_p) on one side, and the thing you don't know (φ) on the other. That means you get rid of the gamma function mess!

$$\phi(\alpha) = \frac{Q_p t_p}{C_v A}$$

All that's left is to determine the relationship between φ and α. This is given by:

$$\alpha = 0.045 + 0.5\phi + 5.6\phi^2 + 0.3\phi^3$$

OK, here's the train of thought so far—use the peak flow, the lag time, and the watershed area to determine the value of φ. Use φ to determine α. Why do this? Here's the last bit. If all this worked, you can determine values of the hydrograph using:

$$Q(t) = Q_p a^\alpha e^{(1-a)\alpha}$$

Where,

$$a = \frac{t}{t_p}$$

Let's try this out! Peak flow rate for the synthetic unit hydrograph of a 36,000 acre watershed is 1720 cfs, and the peak occurs 12 hours after the initiation of runoff. Synthesize the hydrograph.