Lecture 26—Runge-Kutta Techniques

One problem with the Muskingum method is that it *assumes* that the storage equation is linear with depth (that is, it's a simple arrangement of inflow, outflow, and constants...let's talk about what that means). Although this simplifies calculation considerably, it doesn't necessarily have anything to do with reality. A more accurate method would allow for the storage function to be any function of depth. This method (or rather *these* methods) are referred to as the Runge-Kutta methods. Let's have a look

Consider some reservoir or lake. We'd like to know how the flood will be attenuated during its passage through the reservoir (this would work for a river, too, because that's basically a long, skinny lake). The method starts by stating the same continuity equation we're used to:

$$\frac{dS}{dt} = I(t) - O(H)$$

where *H* is the head in the reservoir. Remember head? For standing water (like in a reservoir) *H* is just the depth. So, the storage in the reservoir is a function of the depth and the area, and the change in storage with depth is:

$$\frac{dS}{dH} = A(H)$$

Sooooo, combining these two equations yields:

$$\frac{dH}{dt} = \frac{I(t) - O(H)}{A(H)}$$

or just

$$\frac{dH}{dt} = f(H,t)$$

From here, we can do the same finite differencing technique we did for the Muskingum method, and:

$$\Delta H = \frac{I(t_n) - O(H_n)}{A(H_n)} \Delta t$$

or just

$$\Delta H = f\left(H_n, t_n\right)\Delta t$$

where $H_{n+1} = H_n + \Delta H$

This solution is called the *first-order* Runge-Kutta method (sometimes the Euler method) and is effectively a linear solution for our d*H*/d*t* equation; it'd work *great* if *H* were a linear function of *t*. The problem, though, is that Δ*H* is not constant, but is instead a function of *t* {figure}. This means we introduce error when the true relation between H and t deviates from linear.

One way of solving this problem is to calculate Δ*H* at both the beginning and end of the time interval, and average the two.