

Lecture 5—An interlude on significant figures and a little calculus

Ok, you've all heard about them, you've all DONE them, but somehow we never seem to use them. I'm speaking, of course, of significant figures. Eeeew. Well, they're a fact of life, so let's begin.

Basically, a *significant figure* is a digit that conveys information in a number, rather than a placeholder. For example, in the number 0.0064, the zeros are just holding the places for the actual significant numbers, the 64. Note that in *scientific notation* this number would be compressed to nothing but its significant digits and be written 6.4×10^{-3} . Note also that the zeros *in between* significant figures are also significant, so 0.00604 has three significant digits, not two (and this is borne out by scientific notation, which would write that 6.04×10^{-3} .)

[Practice with significant figures: 0.0000065, 0.00000650, 10.00000650, 0.08000650]

Multiplication & Division

The guiding principle is that you can't have more significant figures in the answer than in the measurements, so if you had, say, a rectangle with sides 680.8 ft and 75.3 ft the area is 51,300 ft², not 51,264.24 ft². Note, however, that this is 1.18 acres, because significant figures are not unit dependent. Back in the day, it was considered a good thing to recalculate significant figures after every computation, simply because it saved time. Now, with a calculator, like you care if you're multiplying 6 into 3 or just 3 into 3. You're better off waiting to the end (this is vital for things like sum-of-squares), but if you have to do it by hand, you might remember the old-school way.

Addition & Subtraction

You'd think this would be simple, right? You'd think it just went like multiplication? Well, it doesn't. I mean, mostly it does, but consider the following: What's $105.2 + 0.000005$? Turns out it's not 105.200005, nor is it 100, though (which would be one significant figure, from the 0.000005). It's (surprise!) 105.2. You continue down to the last digit all of the numbers in

the column share. This ends up making some sense, but as before, remember to add the whole thing up and then determine significant figures to reduce round-off error.

The dreaded calculus

Ok, you've taken it, you may have failed it (I'm with you), but most of you hate it. I'm speaking, of course, of the dreaded calculus. It's amazing how truly much geologists hate calculus. It's especially amazing considering that it's something we take for granted when we speak in words. So, perhaps a little review? At the outset, let me state that I don't really care that you relearn such vagaries as integration by parts, or differentiation in the complex plane. Whatever. I want you to know what a derivative is, what an integral is, and where to go for help integrating and differentiating.

First, what the hell is a derivative? Some of you will say "it's a slope" and others "it's a tangent curve". Basically, a derivative is a measure of a rate of change of a function. This *is* effectively a slope. Look! If you take the function $y=2x$, you could determine the slope, right? It's $\Delta y/\Delta x$, right? This is great. That'd make the slope 2. NOW, suppose we had the function $y=x^2$. The problem here is that it actually depends where you are on the curve what the slope is! Here $\Delta y/\Delta x$ has no meaning unless we determine how big a Δx we're talking about. The bigger it is, the less accurate the slope will be for the point of interest. Conversely, though, the *smaller* Δx is, the more accurate the slope will be. SO, what if we made Δx infinitesimally small? We'd have the most accurate slope, and life would be good. To show that it's incredibly tiny, let's just call it dx instead of Δx , and call dx the *differential* of x .

Now, instead of $\Delta y/\Delta x$ we have dy/dx . What is dy/dx ? (this is a formalism, by the way, saying "the derivative of y with respect to x ". We'll talk about why we need it in a minute.) For our example above, y' here is 2. The nice thing is that the slope doesn't change with location. For any x , the slope is 2. Nice. It doesn't have to be this way, though. You may remember that the derivative of $y=x^2$ is $2x$. This *does* vary with location. No matter what, though, it's a statement of the slope of x^2 at any point x . We don't have to deal with just location, either. Suppose we had some volume, V , and that

volume expanded and contracted so that $V = \sin t$. The change in volume with time, then, is $dV/dt = \cos t$.

A lot of times we'll deal with functions that vary both with location and time. What happens if there's more than one variable? Like, $y = xt^2$? This is why we have the formalism— dy/dx is t^2 , but dy/dt is $2xt$. Basically, you treat the other variables like constants and just derivate. To let you know that there's other variables in there, we change the notation from d to ∂ to show it's a *partial* derivative. Big deal.

Hey, on a side note, there's nothing that says you can't have an equation involving both a function AND its derivative. Like, say,

$$k \left(\frac{dQ}{dt} \right) + Q(t) = I(t)$$

which is a basic statement in hydrology. These are called *differential equations* for obvious reasons, and their solution is far more useful than calculus is, mostly because many natural phenomena obey differential equation rules (like hydrologic continuity, for instance). Differential equations having only one variable (like the above) are called *ordinary* differential equations (or ODEs) and their solution is generally handled in a first class in differential equations. Ones with more than one variable are called *partial* differential equations and are handled in a more advanced class. I recommend both classes to you, but promise there will be no ODE or PDE solving in this class.

So, how to take a derivative? You can memorize a few, but in the end, just learn to use a derivative table, and keep one you know and love on hand. If you spend a lot of time doing derivatives, you might invest the time in learning Maple or Mathematica, but for just a few, use a table! You may already have one—most calculus textbooks have one inside the front cover.

As derivation is to addition, so integration is to subtraction. It's the opposite of a derivative. We think of integration as “the area under a curve” and commonly it is. Another simple way of thinking of it is that we can get the equation for the rate of change, but we *wanted* the original quantity. For example, I might have an expression for velocity with time, but I wanted to know how far I went—enter the integral. My advice—get an integral table

and learn to love it. I will introduce a few oddities and leave it at that. First, there's the same "who are we integrating" thing that we had in derivatives. There's the integral symbol, the function, then the "dx" that says who's getting integrated. This helps if there's more than one variable. Second, note that $y=2x+1$ and $y=2x+\pi$ both have the same derivative, 2. It follows, then, that the integral of 2 is not merely "2x", but could be 2x plus *any* constant. We call this an indefinite integral, and call the proper answer $2x+c$. For *definite* integrals (the ones with the little numbers top and bottom) you just insert the top number for the variable you integrated, and subtract the same thing with the bottom number inserted. To wit:

$$\int_0^t 2x dx = x^2 \Big|_0^t = t^2 - 0^2$$

Notice the loss of the constant, because the constant appears in both parts of the definite integral.

Lastly, there's nothing that says we can't integrate more than once, the same way that we can differentiate more than once. Whereas differentiation often happens more than once with the same variable, integration often happens over two different variables. Consider a surface, $z=2x+y$. This is a continuous surface, so its area is infinite. BUT, we could define a section of it, and ask for the area. I won't talk too much more about it, but the formalism looks like $\iint f(x, y) dx dy$ or more simply $\iint f(x, y) dA$. Either way, it's just saying the area of a surface, or the flux through a surface. Similarly, a *triple* integral is a statement of volume or volume change. This makes an easy statement of continuity:

$$0 = \frac{d}{dt} \iiint \rho dV + \iint \rho \mathbf{V} \cdot d\mathbf{A}$$

This just says that in some system the change in mass in the system has to be balanced by flow into or out of the system through the border. This is a form of the Reynolds Transport Equation, and is really just the same thing as our other continuity equations. [the volume integral is the change in storage with time, so it's dS/dt , and the surface integral is a compacted way of saying inflow and outflow, so those are $I(t)$ and $Q(t)$.]