Definitions:

- **Absolute value equations**: Let $c > 0$. Then

\[
|ax + b| = c \iff ax + b = c \text{ or } ax + b = -c
\]

- **Absolute value inequalities**: For $c > 0$,

\[
|ax + b| \leq c \iff -c \leq ax + b \leq c
\]

\[
|ax + b| \geq c \iff ax + b \geq c \text{ or } ax + b \leq -c
\]

Important Properties:

- **Addition Property of Inequality**: If $a, b,$ and $c$ are real numbers, then

\[
a < b \text{ and } a + c < b + c
\]

are equivalent. (That is, you can add or subtract the same quantity on both sides of the inequality without changing the solution.)

- **Multiplication Property of Inequality**: For all real numbers $a, b,$ and $c$, with $c \neq 0$,

1. $a < b$ and $ac < bc$ are equivalent if $c > 0$.
2. $a < b$ and $ac > bc$ are equivalent if $c < 0$.

(That is, whenever you multiply or divide by a negative number you must reverse or flip the inequality.)

- To solve an absolute value equation or inequality rewrite it without the absolute value using the definitions given above.

- To solve $|ax + b| = |cx + d|$, rewrite as

\[
ax + b = cx + d \text{ or } ax + b = -(cx + d).
\]

Solve these two equations for the answers.

Common Mistakes to Avoid:

- Before rewriting the absolute value equation or inequality, make sure the absolute value is isolated on one side. Do NOT rewrite until the absolute value is isolated.

- When solving $|ax + b| \leq c$, the answer must be written as a three-part inequality. Do NOT break up the answer into two inequalities.

- When solving $|ax + b| \geq c$, the answer must be written as two inequalities. Do NOT combine into one three-part inequality.
PROBLEMS

Solve for $x$ in each of the following equations or inequalities.

1. $|x - 3| = 4$

   Since the absolute value is already isolated, we will rewrite the equation.

   $\begin{align*}
   x - 3 &= 4 \\
   x &= 7
   \end{align*}$

   $\begin{align*}
   x - 3 &= -4 \\
   x &= -1
   \end{align*}$

   $x = 7, \ x = -1$

2. $|2 - 3x| - 5 = 7$

   First, we need to isolate the absolute value.

   $\begin{align*}
   |2 - 3x| - 5 &= 7 \\
   |2 - 3x| &= 12
   \end{align*}$

   Rewriting the expression, we get

   $\begin{align*}
   2 - 3x &= 12 \\
   -3x &= 10 \\
   x &= -\frac{10}{3}
   \end{align*}$

   $\begin{align*}
   2 - 3x &= -12 \\
   -3x &= -14 \\
   x &= \frac{14}{3}
   \end{align*}$

   $x = -\frac{10}{3}, \ x = \frac{14}{3}$

3. $|2x + 3| = 5$

   Since the absolute value is already isolated, we will rewrite the equation.

   $\begin{align*}
   2x + 3 &= 5 \\
   2x &= 2 \\
   x &= 1
   \end{align*}$

   $\begin{align*}
   2x + 3 &= -5 \\
   2x &= -8 \\
   x &= -4
   \end{align*}$

   $x = 1, \ x = -4$

4. $5 - |4x + 1| = 2$

   First, we need to isolate the absolute value.

   $\begin{align*}
   5 - |4x + 1| &= 2 \\
   -|4x + 1| &= -3 \\
   |4x + 1| &= 3
   \end{align*}$

   Rewriting the expression, we get

   $\begin{align*}
   4x + 1 &= 3 \\
   4x &= 2 \\
   x &= \frac{2}{4}
   \end{align*}$

   $\begin{align*}
   4x + 1 &= -3 \\
   4x &= -4 \\
   x &= -1
   \end{align*}$

   $x = \frac{1}{2}, \ x = -1$
5. \(|3x - 7| = |4x + 2|

Rewriting the equation, we get

\[
\begin{align*}
3x - 7 &= -(4x + 2) \\
3x - 7 &= -4x - 2 \\
7x - 7 &= -2 \\
7x &= 5 \\
x &= \frac{5}{7}
\end{align*}
\]

\(x = -9, \quad x = \frac{5}{7}\)

6. \(|2x - 3| < 5\)

Rewriting this inequality, we get

\[
\begin{align*}
-5 &< 2x - 3 < 5 \\
-2 &< 2x < 8 \\
-1 &< x < 4
\end{align*}
\]

\(-1 < x < 4\)

7. \(|3x + 5| \geq 7\)

Rewriting this expression, we get

\[
\begin{align*}
3x + 5 &\geq 7 \\
3x &\geq 2 \\
x &\geq \frac{2}{3}
\end{align*}
\]

\[
\begin{align*}
3x + 5 &\leq -7 \\
3x &\leq -12 \\
x &\leq -4
\end{align*}
\]

\(x \geq \frac{2}{3}, \quad x \leq -4\)

8. \(|2 - 5x| - 3 \leq 9\)

First, we need to isolate the absolute value.

\[
\begin{align*}
|2 - 5x| - 3 &\leq 9 \\
|2 - 5x| &\leq 12
\end{align*}
\]

Rewriting this inequality, we get

\[
\begin{align*}
-12 &\leq 2 - 5x & \leq 12 \\
-14 &\leq -5x & \leq 10 \\
\frac{14}{5} &\geq x & \geq -2
\end{align*}
\]

\(-2 \leq x \leq \frac{14}{5}\)

9. \(5 - |2x + 4| < 1\)

First, we need to isolate the absolute value.

\[
\begin{align*}
5 - |2x + 4| &< 1 \\
-|2x + 4| &< -4
\end{align*}
\]

\(|2x + 4| > 4\)

Rewriting the inequality, we get

\[
\begin{align*}
2x + 4 &> 4 \\
x &> 0
\end{align*}
\]

\[
\begin{align*}
2x + 4 &< -4 \\
x &< -8
\end{align*}
\]

\(x > 0, \quad x < -4\)