Definitions:

- **Absolute value equations:** Let $c > 0$. Then

$$\boxed{|ax+b| = c \quad\iff\quad ax+b=c \quad \text{or} \quad ax+b=-c}$$

- **Absolute value inequalities:** For $c > 0$,

$$\boxed{|ax+b| \le c \quad\iff\quad -c \le ax+b \le c}$$

$$\boxed{|ax+b| \ge c \quad\iff\quad ax+b \ge c \quad \text{or} \quad ax+b \le -c}$$

Important Properties:

- **Addition Property of Inequality:** If a, b, and c are real numbers, then

$$a < b \quad \text{and} \quad a + c < b + c$$

are equivalent. (That is, you can add or subtract the same quantity on both sides of the inequality without changing the solution.)

- **Multiplication Property of Inequality:** For all real numbers a, b, and c, with $c \ne 0$,
  1. $a < b$ and $ac < bc$ are equivalent if $c > 0$.
  2. $a < b$ and $ac > bc$ are equivalent if $c < 0$.

  (That is, whenever you multiply or divide by a negative number you must reverse or flip the inequality.)

- To solve an absolute value equation or inequality rewrite it without the absolute value using the definitions given above.

- To solve $|ax+b| = |cx+d|$, rewrite as

$$ax+b = cx+d \qquad \text{or} \qquad ax+b = -(cx+d).$$

Solve these two equations for the answers.

Common Mistakes to Avoid:

- Before rewriting the absolute value equation or inequality, make sure the absolute value is isolated on one side. Do NOT rewrite until the absolute value is isolated.

- When solving $|ax+b| \le c$, the answer must be written as a three-part inequality. Do NOT break up the answer into two inequalities.

- When solving $|ax+b| \ge c$, the answer must be written as two inequalities. Do NOT combine into one three-part inequality.

PROBLEMS

Solve for x in each of the following equations or inequalities.

1. $|x-3|=4$

Since the absolute value is already isolated, we will rewrite the equation.

$$\begin{aligned} x-3&=4 \\ x&=7 \end{aligned} \qquad \Bigg| \qquad \begin{aligned} x-3&=-4 \\ x&=-1 \end{aligned}$$

$$\boxed{x=7, \quad x=-1}$$

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2. $|2-3x|-5=7$

First, we need to isolate the absolute value.

$$\begin{aligned} |2-3x|-5&=7 \\ |2-3x|&=12 \end{aligned}$$

Rewriting the expression, we get

$$\begin{aligned} 2-3x&=12 \\ -3x&=10 \\ x&=-\frac{10}{3} \end{aligned} \qquad \Bigg| \qquad \begin{aligned} 2-3x&=-12 \\ -3x&=-14 \\ x&=\frac{14}{3} \end{aligned}$$

$$\boxed{x=-\frac{10}{3}, \quad x=\frac{14}{3}}$$

3. $|2x+3|=5$

Since the absolute value is already isolated, we will rewrite the equation.

$$\begin{aligned} 2x+3&=5 \\ 2x&=2 \\ x&=1 \end{aligned} \qquad \Bigg| \qquad \begin{aligned} 2x+3&=-5 \\ 2x&=-8 \\ x&=-4 \end{aligned}$$

$$\boxed{x=1, \quad x=-4}$$

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4. $5-|4x+1|=2$

First, we need to isolate the absolute value.

$$\begin{aligned} 5-|4x+1|&=2 \\ -|4x+1|&=-3 \\ |4x+1|&=3 \end{aligned}$$

Rewriting the expression, we get

$$\begin{aligned} 4x+1&=3 \\ 4x&=2 \\ x&=\frac{2}{4} \\ x&=\frac{1}{2} \end{aligned} \qquad \Bigg| \qquad \begin{aligned} 4x+1&=-3 \\ 4x&=-4 \\ x&=-1 \end{aligned}$$

$$\boxed{x=\frac{1}{2}, \quad x=-1}$$

5. $|3x - 7| = |4x + 2|$

Rewriting the equation, we get

$3x - 7 = 4x + 2$	$3x - 7 = -(4x + 2)$
$-x - 7 = 2$	$3x - 7 = -4x - 2$
$-x = 9$	$7x - 7 = -2$
$x = -9$	$7x = 5$
	$x = \frac{5}{7}$

$$\boxed{x = -9, \quad x = \frac{5}{7}}$$

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6. $|2x - 3| < 5$

Rewriting this inequality, we get

$$-5 < 2x - 3 < 5$$
$$-2 < 2x < 8$$
$$-1 < x < 4$$

$$\boxed{-1 < x < 4}$$

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7. $|3x + 5| \geq 7$

Rewriting this expression, we get

$3x + 5 \geq 7$	$3x + 5 \leq -7$
$3x \geq 2$	$3x \leq -12$
$x \geq \frac{2}{3}$	$x \leq -4$

$$\boxed{x \geq \frac{2}{3}, \quad x \leq -4}$$

8. $|2 - 5x| - 3 \leq 9$

First, we need to isolate the absolute value.

$$|2 - 5x| - 3 \leq 9$$
$$|2 - 5x| \leq 12$$

Rewriting this inequality, we get

$$-12 \leq 2 - 5x \leq 12$$
$$-14 \leq -5x \leq 10$$
$$\tfrac{14}{5} \geq x \geq -2$$

$$\boxed{-2 \leq x \leq \frac{14}{5}}$$

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9. $5 - |2x + 4| < 1$

First, we need to isolate the absolute value.

$$5 - |2x + 4| < 1$$
$$-|2x + 4| < -4$$
$$|2x + 4| > 4$$

Rewriting the inequality, we get

$2x + 4 > 4$	$2x + 4 < -4$
$2x > 0$	$2x < -8$
$x > 0$	$x < -4$

$$\boxed{x > 0, \quad x < -4}$$