ADDITION AND SUBTRACTION OF RATIONAL EXPRESSIONS

Definitions:

- **Rational Expression:** is the quotient of two polynomials. For example,

$$\frac{x}{y}, \qquad \frac{x+1}{3x-2}, \qquad \frac{x^2-3x+4}{x^6-3}$$

  are all rational expressions.

- **Lowest terms:** A rational expression is in lowest terms when the numerator and denominator contain no common factors.

Important Properties:

- **To add or subtract rational expressions**: you MUST have a common denominator. Therefore, factor each denominator first to find a common denominator. Then you can add (or subtract) the terms and simplify.
- Remember that $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.
- To find the common denominator, it is NOT always necessary to multiply all denominators together.
- Remember that $a - b = -(b - a)$.

Common Mistakes to Avoid:

- To add and subtract rational expressions you MUST have a common denominator. Be aware that

$$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a+b}.$$

- When subtracting rational expressions remember to distribute the subtraction sign to every term in the numerator of the fraction that follows it. For example,

$$\frac{x}{x-2} - \frac{x+1}{x-2} = \frac{x-(x+1)}{x-2} = \frac{x-x-1}{x-2} = \frac{-1}{x-2}.$$

- $\dfrac{a}{b+c} \neq \dfrac{a}{b} + \dfrac{a}{c}$.
- You CANNOT obtain a common denominator by adding (or subtracting) the same constant in the numerator and denominator. Therefore,

$$\frac{a}{b} \neq \frac{a+c}{b+c}.$$

PROBLEMS

Perform the indicated operations and simplify.

1. $\dfrac{2x+3}{x+1}+\dfrac{3x+2}{x+1}$

$$\frac{2x+3}{x+1}+\frac{3x+2}{x+1}$$
$$\frac{2x+3+3x+2}{x+1}$$
$$\frac{5x+5}{x+1}$$
$$\frac{5(x+1)}{x+1}$$
$$\boxed{5}$$

2. $\dfrac{3}{10x+15}-\dfrac{5}{12x+18}$

$$\frac{3}{10x+15}-\frac{5}{12x+18}$$
$$\frac{3}{5(2x+3)}-\frac{5}{6(2x+3)}$$
$$\frac{3(6)}{30(2x+3)}-\frac{5(5)}{30(2x+3)}$$
$$\frac{18}{30(2x+3)}-\frac{25}{30(2x+3)}$$
$$\frac{18-25}{30(2x+3)}$$
$$\boxed{\frac{-7}{30(2x+3)}}$$

3. $\dfrac{8}{x-4}+\dfrac{2}{4-x}$

$$\frac{8}{x-4}+\frac{2}{4-x}$$
$$\frac{8}{x-4}+\frac{2}{-1(x-4)}$$
$$\frac{8}{x-4}+\frac{-2}{x-4}$$
$$\frac{8+-2}{x-4}$$
$$\boxed{\frac{6}{x-4}}$$

4. $\dfrac{2x}{x+4}+\dfrac{3}{x-7}$

$$\frac{2x}{x+4}+\frac{3}{x-7}$$
$$\frac{2x(x-7)}{(x+4)(x-7)}+\frac{3(x+4)}{(x+4)(x-7)}$$
$$\frac{2x^2-14x}{(x+4)(x-7)}+\frac{3x+12}{(x+4)(x-7)}$$
$$\frac{2x^2-14x+3x+12}{(x+4)(x-7)}$$
$$\frac{2x^2-11x+12}{(x+4)(x-7)}$$
$$\boxed{\frac{(2x-3)(x-4)}{(x+4)(x-7)}}$$

5. $\dfrac{2}{x+3} - \dfrac{1}{x^2+7x+12}$

$$\frac{2}{x+3} - \frac{1}{x^2+7x+12}$$

$$\frac{2}{x+3} - \frac{1}{(x+4)(x+3)}$$

$$\frac{2(x+4)}{(x+3)(x+4)} - \frac{1}{(x+4)(x+3)}$$

$$\frac{2x+8}{(x+3)(x+4)} - \frac{1}{(x+4)(x+3)}$$

$$\frac{2x+8-1}{(x+3)(x+4)}$$

$$\boxed{\frac{2x+7}{(x+3)(x+4)}}$$

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6. $\dfrac{x}{(x+2)^2} + \dfrac{3}{x+2}$

$$\frac{x}{(x+2)^2} + \frac{3}{x+2}$$

$$\frac{x}{(x+2)(x+2)} + \frac{3(x+2)}{(x+2)(x+2)}$$

$$\frac{x}{(x+2)(x+2)} + \frac{3x+6}{(x+2)(x+2)}$$

$$\frac{x+3x+6}{(x+2)(x+2)}$$

$$\frac{4x+6}{(x+2)(x+2)}$$

$$\boxed{\frac{2(2x+3)}{(x+2)(x+2)}}$$

7. $\dfrac{2}{x-5} - \dfrac{1}{x} - \dfrac{5}{x^2-5x}$

$$\frac{2}{x-5} - \frac{1}{x} - \frac{5}{x^2-5x}$$

$$\frac{2}{x-5} - \frac{1}{x} - \frac{5}{x(x-5)}$$

$$\frac{2x}{x(x-5)} - \frac{(x-5)}{x(x-5)} - \frac{5}{x(x-5)}$$

$$\frac{2x-(x-5)-5}{x(x-5)}$$

$$\frac{2x-x+5-5}{x(x-5)}$$

$$\frac{x}{x(x-5)}$$

$$\boxed{\frac{1}{x-5}}$$

8. $\dfrac{x}{x^2+x-2} - \dfrac{2}{x^2-5x+4}$

$$\frac{x}{x^2+x-2} - \frac{2}{x^2-5x+4}$$

$$\frac{x}{(x+2)(x-1)} - \frac{2}{(x-4)(x-1)}$$

$$\frac{x(x-4)}{(x+2)(x-1)(x-4)} - \frac{2(x+2)}{(x+2)(x-1)(x-4)}$$

$$\frac{x^2-4x}{(x+2)(x-1)(x-4)} - \frac{2x+4}{(x+2)(x-1)(x-4)}$$

$$\frac{x^2-4x-(2x+4)}{(x+2)(x-1)(x-4)}$$

$$\frac{x^2-4x-2x-4}{(x+2)(x-1)(x-4)}$$

$$\boxed{\frac{x^2-6x-4}{(x+2)(x-1)(x-4)}}$$

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9. $\dfrac{4x}{x-1} - \dfrac{2}{x+1} - \dfrac{4}{x^2-1}$

$$\frac{4x}{x-1} - \frac{2}{x+1} - \frac{4}{x^2-1}$$

$$\frac{4x}{x-1} - \frac{2}{x+1} - \frac{4}{(x-1)(x+1)}$$

$$\frac{4x(x+1)}{(x-1)(x+1)} - \frac{2(x-1)}{(x-1)(x+1)} - \frac{4}{(x-1)(x+1)}$$

$$\frac{4x^2+4x}{(x-1)(x+1)} - \frac{2x-2}{(x-1)(x+1)} - \frac{4}{(x-1)(x+1)}$$

$$\frac{4x^2+4x-(2x-2)-4}{(x-1)(x+1)}$$

$$\frac{4x^2+4x-2x+2-4}{(x-1)(x+1)}$$

$$\frac{4x^2+2x-2}{(x-1)(x+1)}$$

$$\frac{2(2x^2+x-1)}{(x-1)(x+1)}$$

$$\frac{2(2x-1)(x+1)}{(x-1)(x+1)}$$

$$\boxed{\frac{2(2x-1)}{x-1}}$$

10. $\dfrac{2x}{x+3} - \dfrac{8}{x^2+8x+15} - \dfrac{4}{x+5}$

$$\frac{2x}{x+3} - \frac{8}{x^2+8x+15} - \frac{4}{x+5}$$

$$\frac{2x}{x+3} - \frac{8}{(x+5)(x+3)} - \frac{4}{x+5}$$

$$\frac{2x(x+5)}{(x+5)(x+3)} - \frac{8}{(x+5)(x+3)} - \frac{4(x+3)}{(x+5)(x+3)}$$

$$\frac{2x^2+10x}{(x+5)(x+3)} - \frac{8}{(x+5)(x+3)} - \frac{4x+12}{(x+5)(x+3)}$$

$$\frac{2x^2+10x-8-(4x+12)}{(x+5)(x+3)}$$

$$\frac{2x^2+10x-8-4x-12}{(x+5)(x+3)}$$

$$\frac{2x^2+6x-20}{(x+5)(x+3)}$$

$$\frac{2(x^2+3x-10)}{(x+5)(x+3)}$$

$$\frac{2(x+5)(x-2)}{(x+5)(x+3)}$$

$$\boxed{\frac{2(x-2)}{x+3}}$$