Definitions:

- **Rational Expression**: is the quotient of two polynomials. For example,

  \[
  \frac{x}{y}, \quad \frac{x + 1}{3x - 2}, \quad \frac{x^2 - 3x + 4}{x^6 - 3}
  \]

  are all rational expressions.

- **Lowest terms**: A rational expression is in lowest terms when the numerator and denominator contain no common factors.

Important Properties:

- **To add or subtract rational expressions**: you MUST have a common denominator. Therefore, factor each denominator first to find a common denominator. Then you can add (or subtract) the terms and simplify.

- **To find the common denominator**, it is NOT always necessary to multiply all denominators together.

- **To subtract rational expressions**, remember to distribute the subtraction sign to every term in the numerator of the fraction that follows it. For example,

  \[
  \frac{x}{x - 2} - \frac{x + 1}{x - 2} = \frac{x - (x + 1)}{x - 2} = \frac{x - x - 1}{x - 2} = \frac{-1}{x - 2}.
  \]

  - \[
  \frac{a}{b + c} \neq \frac{a}{b} + \frac{a}{c}.
  \]

Common Mistakes to Avoid:

- **To add and subtract rational expressions** you MUST have a common denominator. Be aware that

  \[
  \frac{1}{a} + \frac{1}{b} \neq \frac{1}{a + b}.
  \]

- **When subtracting rational expressions** remember to distribute the subtraction sign to every term in the numerator of the fraction that follows it. For example,

  \[
  \frac{x}{x - 2} - \frac{x + 1}{x - 2} = \frac{x - (x + 1)}{x - 2} = \frac{x - x - 1}{x - 2} = \frac{-1}{x - 2}.
  \]

  - \[
  \frac{a}{b} \neq \frac{a + c}{b + c}.
  \]
**PROBLEMS**

Perform the indicated operations and simplify.

1. \(\frac{2x + 3}{x + 1} + \frac{3x + 2}{x + 1}\)

\[
\frac{2x + 3}{x + 1} + \frac{3x + 2}{x + 1} = \frac{2x + 3 + 3x + 2}{x + 1} = \frac{5x + 5}{x + 1} = \frac{5(x + 1)}{x + 1} = 5
\]

2. \(\frac{3}{10x + 15} - \frac{5}{12x + 18}\)

\[
\frac{3}{10x + 15} - \frac{5}{12x + 18} = \frac{3(6)}{30(2x + 3)} - \frac{5(5)}{30(2x + 3)} = \frac{18 - 25}{30(2x + 3)} = \frac{-7}{30(2x + 3)}
\]

3. \(\frac{8}{x - 4} + \frac{2}{4 - x}\)

\[
\frac{8}{x - 4} + \frac{2}{4 - x} = \frac{8}{x - 4} - \frac{2}{x - 4} = \frac{8 - 2}{x - 4} = \frac{6}{x - 4}
\]

4. \(\frac{2x}{x + 4} + \frac{3}{x - 7}\)

\[
\frac{2x}{x + 4} + \frac{3}{x - 7} = \frac{2x(x - 7)}{(x + 4)(x - 7)} + \frac{3(x + 4)}{(x + 4)(x - 7)} = \frac{2x^2 - 14x + 3x + 12}{(x + 4)(x - 7)} = \frac{2x^2 - 11x + 12}{(x + 4)(x - 7)} + \frac{(2x - 3)(x - 4)}{(x + 4)(x - 7)}
\]

\[
\frac{2x^2 - 11x + 12}{(x + 4)(x - 7)} + \frac{(2x - 3)(x - 4)}{(x + 4)(x - 7)} = \frac{(2x - 3)(x - 4) + 2x^2 - 11x + 12}{(x + 4)(x - 7)}
\]

\[
\frac{(2x - 3)(x - 4) + 2x^2 - 11x + 12}{(x + 4)(x - 7)} = \frac{2x^2 - 11x + 12}{(x + 4)(x - 7)} + \frac{(2x - 3)(x - 4)}{(x + 4)(x - 7)}
\]

\[
\frac{2x^2 - 11x + 12}{(x + 4)(x - 7)} + \frac{(2x - 3)(x - 4)}{(x + 4)(x - 7)} = \frac{2x^2 - 11x + 12 + (2x - 3)(x - 4)}{(x + 4)(x - 7)}
\]

\[
\frac{2x^2 - 11x + 12 + (2x - 3)(x - 4)}{(x + 4)(x - 7)} = \frac{2x^2 - 11x + 12 + 2x^2 - 11x + 12}{(x + 4)(x - 7)} = \frac{4x^2 - 22x + 24}{(x + 4)(x - 7)}
\]
5. \[ \frac{2}{x + 3} - \frac{1}{x^2 + 7x + 12} \]

\[ \frac{2}{x + 3} - \frac{1}{x^2 + 7x + 12} = \frac{2}{x + 3} - \frac{1}{(x + 4)(x + 3)} \]

\[ \frac{2(x + 4)}{(x + 3)(x + 4)} - \frac{1}{(x + 4)(x + 3)} = \frac{2(x + 8) - 1}{(x + 3)(x + 4)} \]

\[ \frac{2x + 7}{(x + 3)(x + 4)} \]

7. \[ \frac{2}{x - 5} - \frac{1}{x} - \frac{5}{x^2 - 5x} \]

\[ \frac{2}{x - 5} - \frac{1}{x} - \frac{5}{x(x - 5)} \]

\[ \frac{2x}{x(x - 5)} - \frac{(x - 5)}{x(x - 5)} - \frac{5}{x(x - 5)} \]

\[ \frac{2x}{x(x - 5)} - \frac{(x - 5) - 5}{x(x - 5)} \]

\[ \frac{2x - x + 5 - 5}{x(x - 5)} = \frac{x}{x(x - 5)} \]

\[ \frac{1}{x - 5} \]

6. \[ \frac{x}{(x + 2)^2} + \frac{3}{x + 2} \]

\[ \frac{x}{(x + 2)^2} + \frac{3}{x + 2} = \frac{x}{(x + 2)(x + 2)} + \frac{3(x + 2)}{(x + 2)(x + 2)} \]

\[ \frac{x}{(x + 2)(x + 2)} + \frac{3x + 6}{(x + 2)(x + 2)} = \frac{x + 3x + 6}{(x + 2)(x + 2)} \]

\[ \frac{4x + 6}{(x + 2)(x + 2)} \]

\[ \frac{2(2x + 3)}{(x + 2)(x + 2)} \]
8. \[ \frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} \]

\[ \frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} \]

\[ \frac{x}{(x + 2)(x - 1)} - \frac{2}{(x - 4)(x - 1)} \]

\[ \frac{x(x - 4)}{(x + 2)(x - 1)(x - 4)} - \frac{2(x + 2)}{(x + 2)(x - 1)(x - 4)} \]

\[ \frac{x^2 - 4x}{(x + 2)(x - 1)(x - 4)} - \frac{2x + 4}{(x + 2)(x - 1)(x - 4)} \]

\[ \frac{x^2 - 4x - (2x + 4)}{(x + 2)(x - 1)(x - 4)} \]

\[ \frac{x^2 - 4x - 2x - 4}{(x + 2)(x - 1)(x - 4)} \]

\[ \frac{x^2 - 6x - 4}{(x + 2)(x - 1)(x - 4)} \]

9. \[ \frac{4x}{x - 1} - \frac{2}{x + 1} - \frac{4}{x^2 - 1} \]

\[ \frac{4x}{x - 1} - \frac{2}{x + 1} - \frac{4}{x^2 - 1} \]

\[ \frac{4x(x + 1)}{(x - 1)(x + 1)} - \frac{2(x - 1)}{(x - 1)(x + 1)} - \frac{4}{(x - 1)(x + 1)} \]

\[ \frac{4x^2 + 4x}{(x - 1)(x + 1)} - \frac{2x - 2}{(x - 1)(x + 1)} - \frac{4}{(x - 1)(x + 1)} \]

\[ \frac{4x^2 + 4x - (2x - 2) - 4}{(x - 1)(x + 1)} \]

\[ \frac{4x^2 + 4x - 2x + 2 - 4}{(x - 1)(x + 1)} \]

\[ \frac{4x^2 + 2x - 2}{(x - 1)(x + 1)} \]

\[ \frac{2(2x^2 + x - 1)}{(x - 1)(x + 1)} \]

\[ \frac{2(2x - 1)(x + 1)}{(x - 1)(x + 1)} \]

\[ \frac{2(2x - 1)}{x - 1} \]
\[
\frac{2x}{x + 3} - \frac{8}{x^2 + 8x + 15} - \frac{4}{x + 5} \\
\frac{2x}{x + 3} - \frac{8}{x^2 + 8x + 15} - \frac{4}{x + 5} \\
\frac{2x}{x + 3} - \frac{8}{(x + 5)(x + 3)} - \frac{4}{x + 5} \\
\frac{2x(x + 5)}{(x + 5)(x + 3)} - \frac{8}{(x + 5)(x + 3)} - \frac{4(x + 3)}{(x + 5)(x + 3)} \\
\frac{2x^2 + 10x}{(x + 5)(x + 3)} - \frac{8}{(x + 5)(x + 3)} - \frac{4x + 12}{(x + 5)(x + 3)} \\
\frac{2x^2 + 10x - 8 - (4x + 12)}{(x + 5)(x + 3)} \\
\frac{2x^2 + 10x - 8 - 4x - 12}{(x + 5)(x + 3)} \\
\frac{2x^2 + 6x - 20}{(x + 5)(x + 3)} \\
\frac{2(x^2 + 3x - 10)}{(x + 5)(x + 3)} \\
\frac{2(x + 5)(x - 2)}{(x + 5)(x + 3)} \\
\frac{2(x - 2)}{x + 3}
\]