Definitions:

- **Function**: A function $f$ is a rule that assigns to each element $x$ in the set $A$ exactly one element, called $f(x)$, in the set $B$. The set $A$ is called the domain and the set $B$ is called the range.

- **Domain**: The domain of a function is the set of all real numbers for which the expression is defined as a real number. In other words, it is all the real numbers for which the expression “makes sense.”

Important Properties:

- Remember that you cannot have a zero in the denominator.
- Remember that you cannot have a negative number under an even root.
- Remember that we can evaluate an odd root of a negative number.

Common Mistakes to Avoid:

- Do not exclude from the domain the $x$ values which make the quantity under an odd root negative.

PROBLEMS

Find the domain of each function.

1. $h(x) = \frac{x^3}{x^2 + 2x - 3}$

Since we cannot have a zero in the denominator, we will first find out which numbers make the denominator zero. To do this we will solve $x^2 + 2x - 3 = 0$.

After solving this, we will exclude its solutions from the domain.

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x + 3 = 0$  \hspace{1cm}  $x - 1 = 0$

$x = -3$ \hspace{1cm}  $x = 1$

**Domain:** $x \neq -3, \hspace{0.5cm} x \neq 1$

OR

Domain: All real numbers except $-3$ and $1$
2. \( f(x) = \sqrt{6 - 4x} \)

Remember that we cannot have a negative number under a square root. Therefore, \( 6 - 4x \) must be either positive or zero. Hence,

\[
6 - 4x \geq 0 \\
-4x \geq -6 \\
x \leq \frac{-6}{-4} \\
x \leq \frac{3}{2}
\]

Domain: \( x \leq \frac{3}{2} \)

3. \( f(x) = \frac{x - 1}{3x^2 + 2x - 2} \)

Since we cannot have a zero in the denominator, we will find what values of \( x \) make the denominator zero. In other words, we will solve

\[
3x^2 + 2x - 2 = 0.
\]

Once we find the solution, these numbers will then be excluded from the domain.

Since \( 3x^2 + 2x - 2 \) does not factor, we will use the quadratic formula to solve. Here, \( a = 3 \), \( b = 2 \) and \( c = -2 \).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
= \frac{-2 \pm \sqrt{2^2 - 4(3)(-2)}}{2(3)} \\
= \frac{-2 \pm \sqrt{4 + 24}}{6} \\
= \frac{-2 \pm \sqrt{28}}{6} \\
= \frac{-2 \pm 2\sqrt{7}}{6} \\
= \frac{2(-1 \pm \sqrt{7})}{6} \\
= \frac{-1 \pm \sqrt{7}}{3}
\]

\[
\boxed{\text{Domain: } x \neq \frac{-1 \pm \sqrt{7}}{3}}
\]

4. \( g(x) = \frac{\sqrt{x}}{4x - 7} \)

Since we cannot have zero in the denominator, we will need to find out what value of \( x \) makes \( 4x - 7 \) zero and exclude this from the domain.

\[
4x - 7 = 0 \\
4x = 7 \\
x = \frac{7}{4}
\]

In addition, we are also unable to take the square root of a negative number. Therefore, \( x \) must be positive or zero. In other words, \( x \geq 0 \).

As a result of excluding \( x = \frac{7}{4} \) and insisting that \( x \geq 0 \), we get the following answer.

\[
\boxed{\text{Domain: } x \geq 0, \ x \neq \frac{7}{4}}
\]
5. \( g(x) = \frac{\sqrt[3]{x + 1}}{x^2 - 4} \)

The first thing to note is that since we can evaluate the cube root of a positive, negative, or zero number, we do not need to make any restrictions from the numerator. However, we once again cannot have zero in the denominator. Therefore, we must find the values of \( x \) that make the denominator zero by solving

\[
x^2 - 4 = 0.
\]

Once we have solved this, we will eliminate its solutions from the domain.

\[
x^2 - 4 = 0
x^2 = 4
\sqrt{x^2} = \sqrt{4}
x = \pm 2
\]

Domain: \( x \neq 2, \ x \neq 2 \)

6. \( h(x) = \frac{2x - 3}{\sqrt{x - 7}} \)

For this problem, not only can we not have a negative under the 4-th root, but since the radical occurs in the denominator, we also cannot have a zero under it. Therefore, \( x - 7 > 0 \). Solving this, we get

\[
x - 7 > 0
x > 7
\]

Domain: \( x > 7 \)

7. \( f(x) = \frac{7x + 8}{6x^2 - 19x + 10} \)

We must determine where the denominator is zero. To do this we will solve

\[
6x^2 - 19x + 10 = 0.
\]

Therefore,

\[
(3x - 2)(2x - 5) = 0
\]

\[
3x - 2 = 0 \quad 2x - 5 = 0
3x = 2 \quad 2x = 5
x = \frac{2}{3} \quad x = \frac{5}{2}
\]

Excluding these values from the domain, we get

Domain: \( x \neq \frac{2}{3}, \ x \neq \frac{5}{2} \)
8. \[ g(x) = \frac{3x^2 + 5x - 2}{\sqrt[3]{7 - 3x}} \]

Remember that we are able to evaluate the cube root of a negative number. However, since the cube root is in the denominator, we are not allowed to let it be zero. As a result, we will find where the denominator is zero, and exclude this value from the domain.

\[
7 - 3x = 0 \\
-3x = -7 \\
x = \frac{-7}{-3} \\
x = \frac{7}{3}
\]

\[ \text{Domain: } x \neq \frac{7}{3} \]

9. \[ h(x) = \frac{\sqrt{2 + 7x}}{x^2 - 8x + 7} \]

First, let us deal with the numerator. Because we have a square root, we are unable to take the square root of a negative number. Therefore, we will solve for where \(2 + 7x \geq 0\).

\[
2 + 7x \geq 0 \\
7x \geq -2 \\
x \geq \frac{-2}{7}
\]

Next, we are not allowed to have a zero in the denominator. Therefore, we will solve for where \(x^2 - 8x + 7 = 0\) and then eliminate these values from the domain.

\[
x^2 - 8x + 7 = 0 \\
(x - 7)(x - 1) = 0
\]

10. \[ g(x) = \frac{\sqrt{3x + 2}}{9x - 5} \]

Since we are not able to take the 4th root of a negative number, we need to solve for where \(3x + 2 \geq 0\).

\[
3x + 2 \geq 0 \\
3x \geq -2 \\
x \geq \frac{-2}{3}
\]

Next, the denominator cannot be zero. Hence, we will solve for where \(9x - 5 = 0\) and eliminate this value from the domain.

\[
9x - 5 = 0 \\
9x = 5 \\
x = \frac{5}{9}
\]

Putting these together, we get

\[ \text{Domain: } x \geq \frac{-2}{3}, \; x \neq \frac{5}{9} \]