

**Definition:**

- An **exponent** is a number that tells how many times a factor is repeated in a product. For example, in the problem  $2^4$ , 2 is called the base and 4 is the exponent.

$$2^4 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ times}} = 16.$$

**Integer Exponent Rules:**

- **Product Rule:** For any integers  $m$  and  $n$ ,

$$a^m \cdot a^n = a^{m+n}.$$

When multiplying like bases, we add the exponents.

- **Quotient Rule:** For any nonzero number  $a$  and any integers  $m$  and  $n$ ,

$$\frac{a^m}{a^n} = a^{m-n}.$$

When we divide like bases, we subtract the exponents.

- **Power Rule:** For any integers  $m$  and  $n$ ,

$$(a^m)^n = a^{mn}.$$

When we raise a power to another power, we multiply the exponents.

- For any integer  $m$ ,

$$(ab)^m = a^m \cdot b^m.$$

When we have a product raised to a power, we raise each factor to the power.

- For any integer  $m$ ,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}.$$

When we have a quotient raised to a power, we raise both the numerator and denominator to the power.

- **Zero Exponent Rule:** For any nonzero real number  $a$ ,

$$a^0 = 1.$$

- **Negative Exponents:** For any nonzero real number  $a$  and any integer  $n$ ,

$$a^{-n} = \frac{1}{a^n}.$$

- For any nonzero numbers  $a$  and  $b$ , and any integers  $m$  and  $n$ ,

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}.$$

- For any nonzero numbers  $a$  and  $b$ , and any integers  $m$  and  $n$ ,

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m.$$

### Common Mistakes to Avoid:

- When using the product rule, the bases MUST be the same. If they are not, then the expressions cannot be combined. Also, remember to keep the bases the same and only add the exponents. For example,

$$3^2 \cdot 3^4 = 3^{2+4} = 3^6 \qquad 3^2 \cdot 3^4 \neq 9^6.$$

- When using the quotient rule, the bases MUST be the same. If they are not, then the expressions cannot be combined. Also, remember to keep the bases the same and only subtract the exponents. For example,

$$\frac{4^5}{4^3} = 4^2 = 16.$$

- When using the power rule, remember that ALL factors are raised to the power. This includes any constants. For example,

$$(2x^3y^4)^2 = 2^2(x^3)^2(y^4)^2 = 4x^6y^8.$$

- A positive constant raised to a negative power does NOT yield a negative number. For example,

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}, \qquad 3^{-2} \neq -6.$$

- The Power Rule and Quotient Rule do NOT hold for sums and differences. In other words,

$$(a + b)^m \neq a^m + b^m \qquad \text{and} \qquad (a - b)^m \neq a^m - b^m.$$

We will see later how to simplify these.

## PROBLEMS

Use a combination of the exponent rules to simplify each expression. Write answers with only positive exponents. Assume all variable represent nonzero real numbers.

1.  $(\frac{1}{2}x^4)(16x^5)$

$$\begin{aligned}(\frac{1}{2}x^4)(16x^5) &= \frac{1}{2} \cdot 16x^4x^5 \\ &= 8x^{4+5} \\ &= 8x^9\end{aligned}$$

Answer:  $8x^9$

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2.  $(-3x^{-2})(4x^4)$

$$\begin{aligned}(-3x^{-2})(4x^4) &= -3 \cdot 4x^{-2}x^4 \\ &= -12x^{-2+4} \\ &= -12x^2\end{aligned}$$

Answer:  $-12x^2$

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3.  $\frac{(2x^2)^3}{4x^4}$

$$\begin{aligned}\frac{(2x^2)^3}{4x^4} &= \frac{2^3x^{2 \cdot 3}}{4x^4} \\ &= \frac{8x^6}{4x^4} \\ &= 2x^{6-4} \\ &= 2x^2\end{aligned}$$

Answer:  $2x^2$

4.  $\frac{(2x^3)(3x^2)}{(x^2)^3}$

$$\begin{aligned}\frac{(2x^3)(3x^2)}{(x^2)^3} &= \frac{2 \cdot 3x^3x^2}{x^{2 \cdot 3}} \\ &= \frac{6x^{3+2}}{x^6} \\ &= \frac{6x^5}{x^6} \\ &= 6x^{5-6} \\ &= 6x^{-1} \\ &= \frac{6}{x}\end{aligned}$$

Answer:  $\frac{6}{x}$

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5.  $(\frac{1}{6}a^5)(-3a^2)(4a^7)$

$$\begin{aligned}(\frac{1}{6}a^5)(-3a^2)(4a^7) &= \frac{1}{6} \cdot (-3) \cdot 4a^5a^2a^7 \\ &= \frac{-12}{6}a^{5+2+7} \\ &= -2a^{14}\end{aligned}$$

Answer:  $-2a^{14}$

$$6. \quad \frac{(6x^3)^2}{(2x^2)^3} \cdot (3x^2)^0$$

$$\begin{aligned} \frac{(6x^3)^2}{(2x^2)^3} \cdot (3x^2)^0 &= \frac{6^2 x^{3 \cdot 2}}{2^3 x^{2 \cdot 3}} \cdot 1 \\ &= \frac{36x^6}{8x^6} \\ &= \frac{36}{8} \\ &= \frac{9}{2} \end{aligned}$$

Answer: $\frac{9}{2}$
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$$7. \quad \frac{14x^2y^{-7}}{6x^{-3}y^{-4}}$$

$$\begin{aligned} \frac{14x^2y^{-7}}{6x^{-3}y^{-4}} &= \frac{14x^2x^3y^4}{6y^7} \\ &= \frac{14x^{2+3}y^{4-7}}{6} \\ &= \frac{14x^5y^{-3}}{6} \\ &= \frac{14x^5}{6y^3} \\ &= \frac{7x^5}{3y^3} \end{aligned}$$

Answer: $\frac{7x^5}{3y^3}$
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$$8. \quad (-2x^2y)^2 (2x^{-5}y)^{-3}$$

$$\begin{aligned} (-2x^2y)^2 (2x^{-5}y)^{-3} &= (-2)^2 x^{2 \cdot 2} y^2 2^{-3} x^{-5 \cdot -3} y^{-3} \\ &= 4x^4 y^2 2^{-3} x^{15} y^{-3} \\ &= \frac{4x^4 y^2 x^{15}}{2^3 y^3} \\ &= \frac{4x^{4+15} y^2}{8y^3} \\ &= \frac{x^{19}}{2y} \end{aligned}$$

Answer: $\frac{x^{19}}{2y}$
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$$9. \quad \frac{(xy^2w^{-3})^4}{(x^{-3}y^{-2}w)^3}$$

$$\begin{aligned} \frac{(xy^2w^{-3})^4}{(x^{-3}y^{-2}w)^3} &= \frac{x^4 y^{2 \cdot 4} w^{-3 \cdot 4}}{x^{-3 \cdot 3} y^{-2 \cdot 3} w^3} \\ &= \frac{x^4 y^8 w^{-12}}{x^{-9} y^{-6} w^3} \\ &= \frac{x^4 x^9 y^8 y^6}{w^{12} w^3} \\ &= \frac{x^{4+9} y^{6+8}}{w^{12+3}} \\ &= \frac{x^{13} y^{14}}{w^{15}} \end{aligned}$$

Answer: $\frac{x^{13} y^{14}}{w^{15}}$
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$$10. \left( \frac{-2x^4y^{-4}}{3x^{-1}y^{-2}} \right)^4$$

$$\begin{aligned} \left( \frac{-2x^4y^{-4}}{3x^{-1}y^{-2}} \right)^4 &= \left( \frac{-2x^4xy^2}{3y^4} \right)^4 \\ &= \left( \frac{-2x^{4+1}y^{2-4}}{3} \right)^4 \\ &= \left( \frac{-2x^5y^{-2}}{3} \right)^4 \\ &= \left( \frac{-2x^5}{3y^2} \right)^4 \\ &= \frac{(-2)^4x^{5 \cdot 4}}{3^4y^{2 \cdot 4}} \\ &= \frac{16x^{20}}{81y^8} \end{aligned}$$

Answer: $\frac{16x^{20}}{81y^8}$
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$$11. \left( \frac{-3a^3b^{-5}}{6a^{-2}b^{-2}} \right)^3$$

$$\begin{aligned} \left( \frac{-3a^3b^{-5}}{6a^{-2}b^{-2}} \right)^3 &= \left( \frac{-3a^3a^2b^2}{6b^5} \right)^3 \\ &= \left( \frac{-3a^{3+2}b^{2-5}}{6} \right)^3 \\ &= \left( \frac{-a^5b^{-3}}{2} \right)^3 \\ &= \left( \frac{-a^5}{2b^3} \right)^3 \\ &= \frac{-a^{5 \cdot 3}}{2^3b^{3 \cdot 3}} \\ &= \frac{-a^{15}}{8b^9} \end{aligned}$$

Answer: $\frac{-a^{15}}{8b^9}$
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$$12. \frac{(-2m^{-5}n^2)^3(3m^2)^{-1}}{m^{-2}n^{-4}(m^{-1})^2}$$

$$\begin{aligned} \frac{(-2m^{-5}n^2)^3(3m^2)^{-1}}{m^{-2}n^{-4}(m^{-1})^2} &= \frac{(-2m^{-5}n^2)^3 m^2 n^4 m^2}{(3m^2)} \\ &= \frac{(-2)^3 m^{-5 \cdot 3} n^{2 \cdot 3} m^2 n^4 m^2}{3m^2} \\ &= \frac{-8m^{-15} n^6 m^2 n^4 m^2}{3m^2} \\ &= \frac{-8m^{-15+2+2} n^{6+4}}{3m^2} \\ &= \frac{-8m^{-11} n^{10}}{3m^2} \\ &= \frac{-8n^{10}}{3m^2 m^{11}} \\ &= \frac{-8n^{10}}{3m^{13}} \end{aligned}$$

Answer: $\frac{-8n^{10}}{3m^{13}}$
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