Definition:

- **Linear equation in two variables:** is an equation that can be written as

$$ax + by = c$$

where a, b, and c are real numbers and a and b cannot both be zero.

Three ways to graph a line:

1. Plot points: Choose values for x (or y) and find ordered pairs. Then plot these ordered pairs and connect them with a straight line.
2. Using intercepts: Find the $x-$intercept and $y-$intercept of the linear equation. Plot these two points and connect them with a straight line.
3. Using the slope and $y-$intercept: Recall that placing the equation in slope-intercept form of $y = mx + b$ identifies the slope and the $y-$intercept. Plot the $y-$intercept first and then use the slope, $m = \frac{\text{rise}}{\text{run}}$, to find another point on the graph. Connect these two points with a straight line.

Important Properties:

- The graph of a linear equation in two variables will always be a line.
- The advantage of using the slope and a point to graph a line is that you do not need to have the equation of the line in order to graph it. You only need to know the slope and a point on the graph.
- $x = c$ represents a vertical line at c.
- $y = c$ represents a horizontal line at c.
- The $x-$intercept is found by setting $y = 0$ and solving for x. The $x-$intercept is represented by the ordered pair $(x, 0)$.
- The $y-$intercept is found by setting $x = 0$ and solving for y. The $y-$intercept is represented by the ordered pair $(0, y)$.
- When rise is positive you go up and when rise is negative you go down.
- When run is positive you go to the right and when run is negative you go to the left.
- Although it is true that two points determine a line, it is better to plot at least three points in order to avoid mistakes.

Common Mistakes to Avoid:

- When your slope is negative, remember to include the negative with either the numerator or the denominator NOT both.

PROBLEMS

1. **Graph** $2x + 3y = 6$.

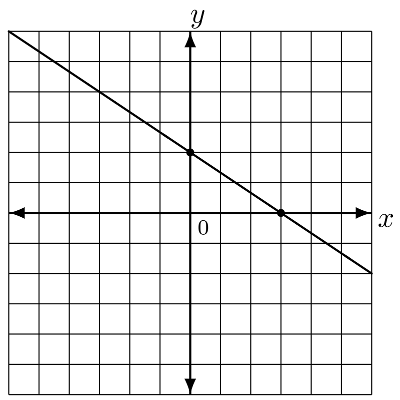
For this problem we will graph the equation using the $x-$ and $y-$intercepts. To find the $x-$intercept we substitute $y = 0$ and find that

$$\begin{aligned} 2x + 3(0) &= 6 \\ 2x &= 6 \\ x &= 3. \end{aligned}$$

For the $y-$intercept we substitute $x = 0$ into the equation and find that

$$\begin{aligned} 2(0) + 3y &= 6 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

Now, plotting the $x-$intercept of $(3, 0)$ and the $y-$intercept of $(0, 2)$ and connecting them with a straight line, we get the following graph of the equation.



2. **Graph** $5x - 2y = 10$.

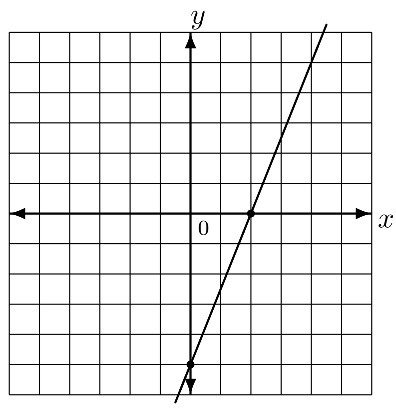
We will graph this line again by finding the $x-$ and $y-$intercepts. To find the $x-$intercept, we let $y = 0$ and find that

$$\begin{aligned} 5x - 2(0) &= 10 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

For the $y-$intercept, we let $x = 0$ and get

$$\begin{aligned} 5(0) - 2y &= 10 \\ -2y &= 10 \\ y &= -5 \end{aligned}$$

Therefore, plotting the intercepts of $(0, -5)$ and $(2, 0)$ and connecting them with a straight line, we get the following graph.



3. **Graph** $3x + 2y = 7$.

We will graph this line by plotting points. Choosing $x = 1$, we find

$$\begin{aligned} 3(1) + 2y &= 7 \\ 3 + 2y &= 7 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

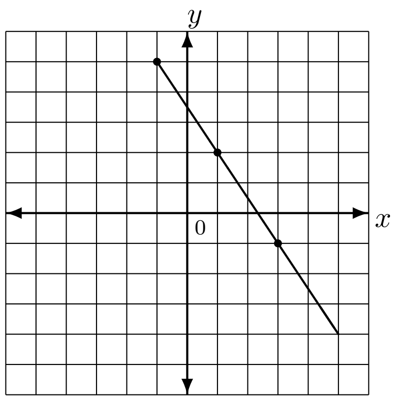
Choosing $x = -1$, we have

$$\begin{aligned} 3(-1) + 2y &= 7 \\ -3 + 2y &= 7 \\ 2y &= 10 \\ y &= 5 \end{aligned}$$

Finally, choosing $x = 3$, we find that

$$\begin{aligned} 3(3) + 2y &= 7 \\ 9 + 2y &= 7 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$

Therefore, when we graph the points $(1, 2)$, $(-1, 5)$, and $(3, -1)$ and connecting them with a straight line, we obtain the following graph.



4. **Graph** $-3x + 4y = 5$.

We will graph this line by plotting points. If we choose $x = -1$, then

$$\begin{aligned} -3(-1) + 4y &= 5 \\ 3 + 4y &= 5 \\ 4y &= 2 \\ y &= \frac{1}{2} \end{aligned}$$

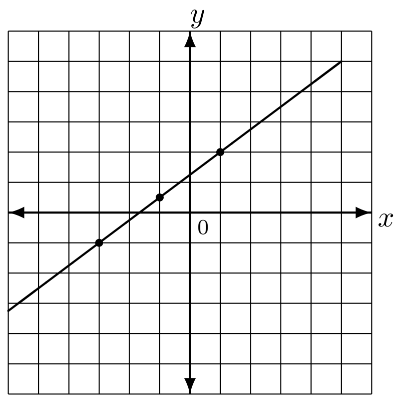
Choosing $x = 1$, we find

$$\begin{aligned} -3(1) + 4y &= 5 \\ -3 + 4y &= 5 \\ 4y &= 8 \\ y &= 2 \end{aligned}$$

Finally, choosing $x = -3$, we have

$$\begin{aligned} -3(-3) + 4y &= 5 \\ 9 + 4y &= 5 \\ 4y &= -4 \\ y &= -1 \end{aligned}$$

Now, plotting the points $\left(-1, \frac{1}{2}\right)$, $(1, 2)$, and $(-3, -1)$ and connecting them with a straight line, we obtain the following graph.

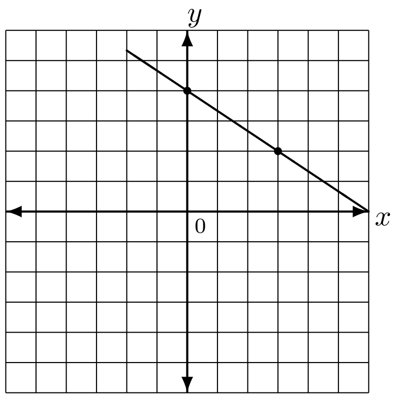


5. **Graph** $2x + 3y = 12$.

We will graph this and the remaining lines using the slope and a point. In order to do this we first need to place the equation in slope-intercept form.

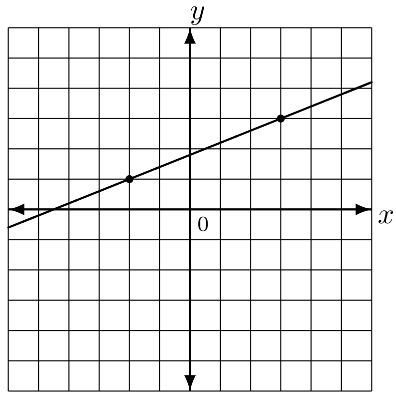
$$\begin{aligned} 2x + 3y &= 12 \\ 3y &= -2x + 12 \\ y &= -\frac{2}{3}x + 4 \end{aligned}$$

Therefore, the $y-$intercept is $(0, 4)$ and the slope $m = -\frac{2}{3}$. So, we will plot the point $(0, 4)$ and then rise -2 (go down 2 units) and run 3 (go right 3 units). This gives us our second point on the graph as $(3, 2)$. Plotting these two points and connecting them with a straight line, we obtain the following graph.



6. **Graph the line with** $m = \frac{2}{5}$ **and which passes through** $(-2, 1)$**.**

We will use the slope and point given to graph this. First, we will plot the point $(-2, 1)$. Next, we will use the slope of $m = \frac{2}{5}$ and rise 2 (go up 2 units) and run 5 (go right 5 units). This gives us our second point at $(3, 3)$. Connecting these points we get the following graph.



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7. **Graph the line with slope** $m = -\frac{3}{2}$ **and passes through** $(-3, -2)$**.**

First, we will plot the point $(-3, -2)$. Then using the slope $m = -\frac{3}{2} = \frac{-3}{2}$, we will rise -3 (go down 3 units) and run 2 (go right 2 units). This gives us our second point at $(-1, -5)$. Connecting these points we will get the following graph.