Definition:

- **Linear equation in two variables**: is an equation that can be written as

  \[ ax + by = c \]

  where \( a, b, \) and \( c \) are real numbers and \( a \) and \( b \) cannot both be zero.

Three ways to graph a line:

1. **Plot points**: Choose values for \( x \) (or \( y \)) and find ordered pairs. Then plot these ordered pairs and connect them with a straight line.

2. **Using intercepts**: Find the \( x \)−intercept and \( y \)−intercept of the linear equation. Plot these two points and connect them with a straight line.

3. **Using the slope and \( y \)−intercept**: Recall that placing the equation in slope-intercept form of \( y = mx + b \) identifies the slope and the \( y \)−intercept. Plot the \( y \)−intercept first and then use the slope, \( m = \frac{\text{rise}}{\text{run}} \), to find another point on the graph. Connect these two points with a straight line.

Important Properties:

- The graph of a linear equation in two variables will always be a line.
- The advantage of using the slope and a point to graph a line is that you do not need to have the equation of the line in order to graph it. You only need to know the slope and a point on the graph.
- \( x = c \) represents a vertical line at \( c \).
- \( y = c \) represents a horizontal line at \( c \).
- The \( x \)−intercept is found by setting \( y = 0 \) and solving for \( x \). The \( x \)−intercept is represented by the ordered pair \((x,0)\).
- The \( y \)−intercept is found by setting \( x = 0 \) and solving for \( y \). The \( y \)−intercept is represented by the ordered pair \((0,y)\).
- When rise is positive you go up and when rise is negative you go down.
- When run is positive you go to the right and when run is negative you go to the left.
- Although it is true that two points determine a line, it is better to plot at least three points in order to avoid mistakes.

Common Mistakes to Avoid:

- When your slope is negative, remember to include the negative with either the numerator or the denominator NOT both.
PROBLEMS

1. Graph \(2x + 3y = 6\).

For this problem we will graph the equation using the \(x\)– and \(y\)–intercepts. To find the \(x\)–intercept we substitute \(y = 0\) and find that

\[
2x + 3(0) = 6 \\
2x = 6 \\
x = 3.
\]

For the \(y\)–intercept we substitute \(x = 0\) into the equation and find that

\[
2(0) + 3y = 6 \\
3y = 6 \\
y = 2.
\]

Now, plotting the \(x\)–intercept of \((3, 0)\) and the \(y\)–intercept of \((0, 2)\) and connecting them with a straight line, we get the following graph of the equation.

2. Graph \(5x - 2y = 10\).

We will graph this line again by finding the \(x\)– and \(y\)–intercepts. To find the \(x\)–intercept, we let \(y = 0\) and find that

\[
5x - 2(0) = 10 \\
5x = 10 \\
x = 2
\]

For the \(y\)–intercept, we let \(x = 0\) and get

\[
5(0) - 2y = 10 \\
-2y = 10 \\
y = -5.
\]

Therefore, plotting the intercepts of \((0, -5)\) and \((2, 0)\) and connecting them with a straight line, we get the following graph.
3. **Graph** $3x + 2y = 7$.

We will graph this line by plotting points. Choosing $x = 1$, we find

\[
3(1) + 2y = 7 \\
3 + 2y = 7 \\
2y = 4 \\
y = 2
\]

Choosing $x = -1$, we have

\[
3(-1) + 2y = 7 \\
-3 + 2y = 7 \\
2y = 10 \\
y = 5
\]

Finally, choosing $x = 3$, we find that

\[
3(3) + 2y = 7 \\
9 + 2y = 7 \\
2y = -2 \\
y = -1
\]

Therefore, when we graph the points $(1, 2)$, $(-1, 5)$, and $(3, -1)$ and connecting them with a straight line, we obtain the following graph.

4. **Graph** $-3x + 4y = 5$.

We will graph this line by plotting points. If we choose $x = -1$, then

\[
-3(-1) + 4y = 5 \\
3 + 4y = 5 \\
4y = 2 \\
y = \frac{1}{2}
\]

Choosing $x = 1$, we find

\[
-3(1) + 4y = 5 \\
-3 + 4y = 5 \\
4y = 8 \\
y = 2
\]

Finally, choosing $x = -3$, we have

\[
-3(-3) + 4y = 5 \\
9 + 4y = 5 \\
4y = -4 \\
y = -1
\]

Now, plotting the points $(-1, \frac{1}{2})$, $(1, 2)$, and $(-3, -1)$ and connecting them with a straight line, we obtain the following graph.
5. **Graph** $2x + 3y = 12$.

We will graph this and the remaining lines using the slope and a point. In order to do this we first need to place the equation in slope-intercept form.

\[
2x + 3y = 12
\]
\[
3y = -2x + 12
\]
\[
y = -\frac{2}{3}x + 4
\]

Therefore, the $y$-intercept is $(0, 4)$ and the slope $m = -\frac{2}{3}$. So, we will plot the point $(0, 4)$ and then rise $-2$ (go down 2 units) and run $3$ (go right 3 units). This gives us our second point on the graph as $(3, 2)$. Plotting these two points and connecting them with a straight line, we obtain the following graph.

6. **Graph the line with** $m = \frac{2}{5}$ **and which passes through** $(−2, 1)$.

We will use the slope and point given to graph this. First, we will plot the point $(-2, 1)$. Next, we will use the slope of $m = \frac{2}{5}$ and rise 2 (go up 2 units) and run 5 (go right 5 units). This gives us our second point at $(3, 3)$. Connecting these points we get the following graph.

7. **Graph the line with slope** $m = -\frac{3}{2}$ **and passes through** $(−3, −2)$.

First, we will plot the point $(-3, -2)$. Then using the slope $m = -\frac{3}{2} = -\frac{3}{2}$, we will rise $-3$ (go down 3 units) and run 2 (go right 2 units). This gives us our second point at $(-1, -5)$. Connecting these points we will get the following graph.