Definition:

- A linear inequality in one variable can be written in the form

$$ax + b < c,$$

where a, b, and c are real numbers. (NOTE: Definition also holds for $>$, $\geq$, $\leq$.)

Important Properties:

- **Addition Property of Inequality:** If a, b, and c are real numbers, then

$$a < b \quad \text{and} \quad a + c < b + c$$

are equivalent. (That is, you can add or subtract the same quantity on both sides of the inequality without changing the solution.)

- **Multiplication Property of Inequality:** For all real numbers a, b, and c, with $c \neq 0$,

1. $a < b$ and $ac < bc$ are equivalent if $c > 0$.

2. $a < b$ and $ac > bc$ are equivalent if $c < 0$.

(That is, whenever you multiply or divide by a negative number you must reverse or flip the inequality.)

Common Mistakes to Avoid:

- DO NOT reverse the inequality when you add or subtract a negative number; only when you multiply or divide by a negative number.

- When clearing the parentheses in an expression like $7 - (2x - 4)$, remember that the minus sign acts like a factor of -1. After using the distributive property, the sign of *every* term in the parentheses will be changed to give $7 - 2x + 4$.

- To clear fractions from an inequality, multiply every term on each side by the lowest common denominator. Remember that $\frac{3x}{2}(x - 2)$ is considered one term, whereas, $\frac{3x^2}{2} - 3x$ is considered two terms. To avoid a mistake, clear all parentheses using the distributive property *before* multiplying every term by the common denominator.

- To preserve the solution to an inequality, remember to perform the same operation on **both** sides (or all parts) of the inequality.

PROBLEMS

Solve for x in each of the following inequalities:

1. $2x - 3 \leq 6 - 5x$

$$\begin{aligned} 2x - 3 &\leq 6 - 5x \\ 7x - 3 &\leq 6 \\ 7x &\leq 9 \\ x &\leq \frac{9}{7} \end{aligned}$$

$$\boxed{x \leq \frac{9}{7}}$$

2. $3(2x + 5) > 4x + 1$

$$\begin{aligned} 3(2x + 5) &> 4x + 1 \\ 6x + 15 &> 4x + 1 \\ 2x + 15 &> 1 \\ 2x &> -14 \\ x &> -7 \end{aligned}$$

$$\boxed{x > -7}$$

3. $2(3 - x) + 1 \leq 4 - (x + 1)$

$$\begin{aligned} 2(3 - x) + 1 &\leq 4 - (x + 1) \\ 6 - 2x + 1 &\leq 4 - x - 1 \\ 7 - 2x &\leq 3 - x \\ 7 - x &\leq 3 \\ -x &\leq -4 \\ x &\geq 4 \end{aligned}$$

$$\boxed{x \geq 4}$$

4. $-(5 + 2x) - 3 + 7x \leq 3(x - 2)$

$$\begin{aligned} -(5 + 2x) - 3 + 7x &\leq 3(x - 2) \\ -5 - 2x - 3 + 7x &\leq 3x - 6 \\ 5x - 8 &\leq 3x - 6 \\ 2x - 8 &\leq -6 \\ 2x &\leq 2 \\ x &\leq 1 \end{aligned}$$

$$\boxed{x \leq 1}$$

5. $\dfrac{5x - 2}{3} > 4$

NOTE: Multiplying each term by the lowest common denominator of 3 will eliminate all fractions.

$$\begin{aligned} \frac{5x - 2}{3} &> 4 \\ 3\left(\frac{5x - 2}{3}\right) &> 3(4) \\ 5x - 2 &> 12 \\ 5x &> 14 \\ x &> \frac{14}{5} \end{aligned}$$

$$\boxed{x > \frac{14}{5}}$$

6. $-\frac{1}{5}(2x+3) < \frac{2}{3}(x-2)$

NOTE: Multiplying each term by the lowest common denominator of 15 will eliminate all fractions.

$$\begin{gathered}
-\frac{1}{5}(2x+3) < \frac{2}{3}(x-2) \\
-\frac{2x}{5} - \frac{3}{5} < \frac{2x}{3} - \frac{4}{3} \\
15\left(-\frac{2x}{5}\right) - 15\left(\frac{3}{5}\right) < 15\left(\frac{2x}{3}\right) - 15\left(\frac{4}{3}\right) \\
-6x - 9 < 10x - 20 \\
-16x - 9 < -20 \\
-16x < -11 \\
x > \frac{11}{16}
\end{gathered}$$

$$\boxed{x > \frac{11}{16}}$$

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7. $3 \leq 2x - 5 < 5$

$$\begin{gathered}
3 \leq 2x - 5 < 5 \\
8 \leq 2x < 10 \\
4 \leq x < 5
\end{gathered}$$

$$\boxed{4 \leq x < 5}$$

8. $-3 < \frac{2-3x}{5} \leq 2$

NOTE: Multiplying each term by the lowest common denominator of 5 will eliminate all fractions.

$$\begin{gathered}
-3 < \frac{2-3x}{5} \leq 2 \\
-3 < \frac{2}{5} - \frac{3x}{5} \leq 2 \\
5(-3) < 5\left(\frac{2}{5}\right) - 5\left(\frac{3x}{5}\right) \leq 5(2) \\
-15 < 2 - 3x \leq 10 \\
-17 < -3x \leq 8 \\
\frac{17}{3} > x \geq -\frac{8}{3}
\end{gathered}$$

$$\boxed{\frac{17}{3} > x \geq -\frac{8}{3}}$$

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9. $-4 < 3(2x-1) + 1 < 8$

$$\begin{gathered}
-4 < 3(2x-1) + 1 < 8 \\
-4 < 6x - 3 + 1 < 8 \\
-4 < 6x - 2 < 8 \\
-2 < 6x < 10 \\
-\frac{2}{6} < x < \frac{10}{6} \\
-\frac{1}{3} < x < \frac{5}{3}
\end{gathered}$$

$$\boxed{-\frac{1}{3} < x < \frac{5}{3}}$$