Definitions:

- **Logarithmic function**: Let a be a positive number with $a \neq 1$. The logarithmic function with base a, denoted $\log_a x$, is defined by

$$\boxed{y = \log_a x \qquad \text{if and only if} \qquad x = a^y.}$$

- **Common logarithm**: The logarithm with base 10 is called the common logarithm. The base 10 is usually omitted when working with the common logarithm.

$$\boxed{\log_{10} x = \log x.}$$

- **Natural logarithm**: The logarithm with base e is called the natural logarithm and is denoted by

$$\boxed{\log_e x = \ln x.}$$

Properties of logarithms: Let a be a positive number such that $a \neq 1$. Then

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a a^x = x$
- $a^{\log_a x} = x$

Other Important Properties:

- $y = \log_a x$ is read: "y is the exponent you place on a in order to get x." A logarithm is an exponent.
- The definition of the logarithm allows us to switch from the logarithmic form of $y = \log_a x$ to the exponential form of $x = a^y$ and back again.
- A logarithmic function is the inverse of an exponential function. An exponential function is the inverse of a logarithmic function.
- You cannot take the logarithm of a negative number or zero.

Properties of the graph of $y = \log_a x$:

- Domain is $(0, \infty)$.
- Range is all real numbers.
- Always crosses through the point $(1, 0)$.
- $x = 0$ is a vertical asymptote.
- If $a > 1$, then the function is increasing; if $0 < a < 1$, then the function is decreasing.

PROBLEMS

1. **Express the equation in logarithmic form:**

(a) $2^5 = 32$

Here, $a = 2$, $y = 5$ and $x = 32$. Thus,

$$\boxed{5 = \log_2 32}$$

(b) $3^{-3} = \frac{1}{27}$

Here, $a = 3$, $y = -3$ and $x = \frac{1}{27}$. Therefore,

$$\boxed{-3 = \log_3 \frac{1}{27}}$$

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2. **Express the equation in exponential form:**

(a) $-2 = \log_7 \frac{1}{49}$

Here, $a = 7$, $y = -2$ and $x = \frac{1}{49}$. Therefore,

$$\boxed{7^{-2} = \frac{1}{49}}$$

(b) $3 = \log_2 8$

Here, $a = 2$, $y = 3$ and $x = 8$. Thus,

$$\boxed{2^3 = 8}$$

3. **Solve for the missing variable.**

(a) $\log_2 16 = y$

$$\log_2 16 = y$$
$$2^y = 16$$
$$2^y = 2^4$$
$$y = 4$$

$$\boxed{\log_2 16 = 4}$$

(b) $\log_3 \frac{1}{81} = y$

$$\log_3 \frac{1}{81} = y$$
$$3^y = \frac{1}{81}$$
$$3^y = \frac{1}{3^4}$$
$$3^y = 3^{-4}$$
$$y = -4$$

$$\boxed{\log_3 \frac{1}{81} = -4}$$

(c) $\log_8 2 = y$

$$\begin{aligned}
\log_8 2 &= y \\
8^y &= 2 \\
8^y &= 2 \\
(2^3)^y &= 2 \\
2^{3y} &= 2^1 \\
3y &= 1 \\
y &= \frac{1}{3}
\end{aligned}$$

$$\boxed{\log_8 2 = \frac{1}{3}}$$

(d) $\log_a \frac{1}{25} = -2$

$$\begin{aligned}
\log_a \frac{1}{25} &= -2 \\
a^{-2} &= \frac{1}{25} \\
\frac{1}{a^2} &= \frac{1}{25} \\
a^2 &= 25 \\
\sqrt{a^2} &= \sqrt{25} \\
a &= 5
\end{aligned}$$

NOTE: a must be a positive number.

$$\boxed{\log_5 \frac{1}{25} = -2}$$

(e) $\log_a 8 = \frac{3}{2}$

$$\begin{aligned}
\log_a 8 &= \frac{3}{2} \\
a^{3/2} &= 8 \\
\left(a^{3/2}\right)^{2/3} &= 8^{2/3} \\
a &= \left(\sqrt[3]{8}\right)^2 \\
a &= 2^2 \\
a &= 4
\end{aligned}$$

$$\boxed{\log_4 8 = \frac{3}{2}}$$

(f) $\log_3 x = 2$

$$\begin{aligned}
\log_3 &= 2 \\
x &= 3^2 \\
x &= 9
\end{aligned}$$

$$\boxed{\log_3 9 = 2}$$

4. **Find the domain of each function.**

(a) $f(x) = \log_3(2x - 1)$

Since we cannot take the logarithm of a negative number or zero, $2x - 1$ must be a positive number. Hence,

$$2x - 1 > 0$$

$$2x > 1$$

$$x > \frac{1}{2}$$

$$\boxed{\text{Domain: } x > \frac{1}{2}}$$

(b) $f(x) = \log_5(5 - 3x)$

Since we cannot take the logarithm of a negative number or zero, $5 - 3x$ must be positive. Hence,

$$5 - 3x > 0$$

$$-3x > -5$$

$$x < \frac{5}{3}$$

$$\boxed{\text{Domain: } x < \frac{5}{3}}$$