MULTIPLICATION AND DIVISION OF RATIONAL EXPRESSIONS

Definitions:

- **Rational Expression:** is the quotient of two polynomials. For example,
$$\frac{x}{y}, \qquad \frac{x+1}{3x-2}, \qquad \frac{x^2-3x+4}{x^6-3}$$
are all rational expressions.
- **Lowest terms:** A rational expression is in lowest terms when the numerator and denominator contain no common factors.
- **Reciprocal:** The reciprocal of a rational expression $\frac{a}{b}$ is given by $\frac{b}{a}$. To find the reciprocal, we invert (or flip) the rational expression.

Important Properties:

- **Fundamental Property of Rational Numbers:** If $\frac{a}{b}$ is a rational number and c is any nonzero real number, then
$$\frac{ac}{bc} = \frac{a}{b}.$$
We use this property to write rational expressions in lowest terms.
- **To write a rational expression in lowest terms:** Factor both the numerator and denominator completely. Apply the Fundamental Property of Rational Numbers to eliminate the common factors.
- **To multiply rational expressions:** Factor all numerators and denominators as much as possible. Apply the Fundamental Property of Rational Numbers to eliminate the common factors. Multiply remaining factors.
- **To divide rational expressions:** Invert the second rational expression and multiply. In other words, multiply the first rational expression by the reciprocal of the second.
- If the numerator and denominator of a rational expression are opposites then the answer is -1. This is because if we factor out a -1 from either the numerator or denominator, we have
$$\frac{x-y}{y-x} = \frac{-1(y-x)}{y-x} = -1.$$

Common Mistakes to Avoid:

- Remember that only common *factors* can be divided out. For example,
$$\frac{(x-2)(x-1)}{(x+4)(x-1)} = \frac{x-2}{x+4}; \quad \text{however,} \quad \frac{x+2}{x+5} \neq \frac{2}{5} \quad \text{and} \quad \frac{x+2}{x} \neq 2.$$
- To multiply (or divide) rational expressions, you do NOT need a common denominator.
- $x+y$ and $y+x$ are NOT opposites of one another. Recall that is does not matter the order in which we add terms together. As a result,
$$\frac{x+y}{y+x} = 1.$$

PROBLEMS

Simplify each expression in lowest terms.

1. $\dfrac{x^2+5x+6}{x^2-2x+15}$

$$\frac{x^2+5x+6}{x^2-2x+15}$$

$$\frac{(x+3)(x+2)}{(x-5)(x+3)}$$

$$\boxed{\frac{(x+2)}{(x-5)}}$$

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2. $\dfrac{y^2-3y-18}{2x^2+7y+3}$

$$\frac{y^2-3y-18}{2y^2+7y+3}$$

$$\frac{(y-6)(y+3)}{(2y+1)(y+3)}$$

$$\boxed{\frac{y-6}{2y+1}}$$

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3. $\dfrac{x^2-1}{1-x^3}$

$$\frac{x^2-1}{1-x^3}$$

$$\frac{(x-1)(x+1)}{(1-x)(1+x+x^2)}$$

$$\frac{-1(1-x)(x+1)}{(1-x)(1+x+x^2)}$$

$$\boxed{\frac{-(x+1)}{1+x+x^2}}$$

4. $\dfrac{6-x-2x^2}{6x^2+x-15}$

$$\frac{6-x-2x^2}{6x^2+x-15}$$

$$\frac{(3-2x)(2+x)}{(2x-3)(3x+5)}$$

$$\frac{-1(2x-3)(2+x)}{(2x-3)(3x+5)}$$

$$\boxed{\frac{-1(2+x)}{3x+5}}$$

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5. $\dfrac{x^2-25}{x^2-x-20}\cdot\dfrac{x^2+x-12}{x^2-5x+6}$

$$\frac{x^2-25}{x^2-x-20}\cdot\frac{x^2+x-12}{x^2-5x+6}$$

$$\frac{(x-5)(x+5)}{(x-5)(x+4)}\cdot\frac{(x+4)(x-3)}{(x-3)(x-2)}$$

$$\boxed{\frac{x+5}{x-2}}$$

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6. $\dfrac{x^2-7x-30}{x^2-6x-40}\cdot\dfrac{2x^2+5x+2}{2x^2+7x+3}$

$$\frac{x^2-7x-30}{x^2-6x-40}\cdot\frac{2x^2+5x+2}{2x^2+7x+3}$$

$$\frac{(x-10)(x+3)}{(x-10)(x+4)}\cdot\frac{(2x+1)(x+2)}{(2x+1)(x+3)}$$

$$\boxed{\frac{x+2}{x+4}}$$

7. $\dfrac{6x^2+5xy-6y^2}{12x^2-11xy+2y^2}\cdot\dfrac{8x^2-14xy+3y^2}{4x^2-12xy+9y^2}$

$$\frac{6x^2+5xy-6y^2}{12x^2-11xy+2y^2}\cdot\frac{8x^2-14xy+3y^2}{4x^2-12xy+9y^2}$$

$$\frac{(2x+3y)(3x-2y)}{(4x-y)(3x-2y)}\cdot\frac{(2x-3y)(4x-y)}{(2x-3y)(2x-3y)}$$

$$\boxed{\frac{2x+3y}{2x-3y}}$$

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8. $\dfrac{y^2+4y-21}{y^2+3y-28}\div\dfrac{y^2+14y+48}{y^2+4y-32}$

$$\frac{y^2+4y-21}{y^2+3y-28}\div\frac{y^2+14y+48}{y^2+4y-32}$$

$$\frac{y^2+4y-21}{y^2+3y-28}\cdot\frac{y^2+4y-32}{y^2+14y+48}$$

$$\frac{(y+7)(y-3)}{(y+7)(y-4)}\cdot\frac{(y+8)(y-4)}{(y+6)(y+8)}$$

$$\boxed{\frac{y-3}{y+6}}$$

9. $\dfrac{6x^4-15x^3}{4x^2-6x}\div\dfrac{45x-18x^2}{2x^2-x-3}$

$$\frac{6x^4-15x^3}{4x^2-6x}\div\frac{45x-18x^2}{2x^2-x-3}$$

$$\frac{6x^4-15x^3}{4x^2-6x}\cdot\frac{2x^2-x-3}{45x-18x^2}$$

$$\frac{3x^3(2x-5)}{2x(2x-3)}\cdot\frac{(2x-3)(x+1)}{9x(5-2x)}$$

$$\frac{-3x^3(5-2x)}{2x(2x-3)}\cdot\frac{(2x-3)(x+1)}{9x(5-2x)}$$

$$\boxed{\frac{-x(x+1)}{6}}$$

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10. $\dfrac{3x^2y^2-12x^2}{y^2+10y+16}\div\dfrac{xy^2-12xy+35x}{y^2+3y-40}$

$$\frac{3x^2y^2-12x^2}{y^2+10y+16}\div\frac{xy^2-12xy+35x}{y^2+3y-40}$$

$$\frac{3x^2y^2-12x^2}{y^2+10y+16}\cdot\frac{y^2+3y-40}{xy^2-12xy+35x}$$

$$\frac{3x^2(y^2-4)}{(y+8)(y+2)}\cdot\frac{(y+8)(y-5)}{x(y^2-12y+35)}$$

$$\frac{3x^2(y-2)(y+2)}{(y+8)(y+2)}\cdot\frac{(y+8)(y-5)}{x(y-7)(y-5)}$$

$$\boxed{\frac{3x(y-2)}{y-7}}$$