Definitions:

- **Rational Expression**: is the quotient of two polynomials. For example,
  \[ \frac{x}{y}, \quad \frac{x + 1}{3x - 2}, \quad \frac{x^2 - 3x + 4}{x^6 - 3} \]
  are all rational expressions.

- **Lowest terms**: A rational expression is in lowest terms when the numerator and denominator contain no common factors.

- **Reciprocal**: The reciprocal of a rational expression \( \frac{a}{b} \) is given by \( \frac{b}{a} \). To find the reciprocal, we invert (or flip) the rational expression.

Important Properties:

- **Fundamental Property of Rational Numbers**: If \( \frac{a}{b} \) is a rational number and \( c \) is any nonzero real number, then
  \[ \frac{ac}{bc} = \frac{a}{b}. \]
  We use this property to write rational expressions in lowest terms.

- **To write a rational expression in lowest terms**: Factor both the numerator and denominator completely. Apply the Fundamental Property of Rational Numbers to eliminate the common factors.

- **To multiply rational expressions**: Factor all numerators and denominators as much as possible. Apply the Fundamental Property of Rational Numbers to eliminate the common factors. Multiply remaining factors.

- **To divide rational expressions**: Invert the second rational expression and multiply. In other words, multiply the first rational expression by the reciprocal of the second.

- If the numerator and denominator of a rational expression are opposites then the answer is \(-1\). This is because if we factor out a \(-1\) from either the numerator or denominator, we have
  \[ \frac{x - y}{y - x} = \frac{-1(y - x)}{y - x} = -1. \]

Common Mistakes to Avoid:

- Remember that only common factors can be divided out. For example,
  \[ \frac{(x - 2)(x - 1)}{(x + 4)(x - 1)} = \frac{x - 2}{x + 4}; \quad \text{however, } \frac{x + 2}{x + 5} \neq \frac{2}{5} \quad \text{and} \quad \frac{x + 2}{x} \neq 2. \]

- To multiply (or divide) rational expressions, you do NOT need a common denominator.

- \( x + y \) and \( y + x \) are NOT opposites of one another. Recall that is does not matter the order in which we add terms together. As a result,
  \[ \frac{x + y}{y + x} = 1. \]
PROBLEMS

Simplify each expression in lowest terms.

1. \( \frac{x^2 + 5x + 6}{x^2 - 2x + 15} \)
   \[\frac{x^2 + 5x + 6}{x^2 - 2x + 15} \frac{(x + 3)(x + 2)}{(x - 5)(x + 3)} \]
   \[\frac{(x + 2)}{(x - 5)}\]

2. \( \frac{y^2 - 3y - 18}{2x^2 + 7y + 3} \)
   \[\frac{y^2 - 3y - 18}{2y^2 + 7y + 3} \frac{(y - 6)(y + 3)}{(2y + 1)(y + 3)} \]
   \[\frac{y - 6}{2y + 1}\]

3. \( \frac{x^2 - 1}{1 - x^3} \)
   \[\frac{x^2 - 1}{1 - x^3} \frac{(x - 1)(x + 1)}{(1 - x)(1 + x + x^2)} \]
   \[\frac{-1(1 - x)(x + 1)}{(1 - x)(1 + x + x^2)} \]
   \[\frac{-(x + 1)}{1 + x + x^2}\]

4. \( \frac{6 - x - 2x^2}{6x^2 + x - 15} \)
   \[\frac{6 - x - 2x^2}{6x^2 + x - 15} \frac{(3 - 2x)(2 + x)}{(2x - 3)(3x + 5)} \]
   \[\frac{-1(2x - 3)(2 + x)}{(2x - 3)(3x + 5)} \]
   \[\frac{-1(2 + x)}{3x + 5}\]

5. \( \frac{x^2 - 25}{x^2 - x - 20} \cdot \frac{x^2 + x - 12}{x^2 - 5x + 6} \)
   \[\frac{x^2 - 25}{x^2 - x - 20} \cdot \frac{x^2 + x - 12}{x^2 - 5x + 6} \frac{(x - 5)(x + 5)}{(x - 5)(x + 4)} \]
   \[\frac{(x + 4)(x - 3)}{(x - 3)(x - 2)} \]
   \[\frac{x + 5}{x - 2}\]

6. \( \frac{x^2 - 7x - 30}{x^2 - 6x - 40} \cdot \frac{2x^2 + 5x + 2}{2x^2 + 7x + 3} \)
   \[\frac{x^2 - 7x - 30}{x^2 - 6x - 40} \cdot \frac{2x^2 + 5x + 2}{2x^2 + 7x + 3} \]
   \[\frac{(x - 10)(x + 3)}{(x - 10)(x + 4)} \cdot \frac{(2x + 1)(x + 2)}{(2x + 1)(x + 3)} \]
   \[\frac{x + 2}{x + 4}\]
7. \[
\frac{6x^2 + 5xy - 6y^2}{12x^2 - 11xy + 2y^2} \cdot \frac{8x^2 - 14xy + 3y^2}{4x^2 - 12xy + 9y^2}
\]
\[
\frac{6x^2 + 5xy - 6y^2}{12x^2 - 11xy + 2y^2} \cdot \frac{8x^2 - 14xy + 3y^2}{4x^2 - 12xy + 9y^2}
\]
\[
(2x + 3y)(3x - 2y) \quad (2x - 3y)(4x - y) \quad (2x - 3y)(2x - 3y)
\]
\[
\frac{2x + 3y}{2x - 3y}
\]

8. \[
\frac{y^2 + 4y - 21}{y^2 + 3y - 28} \div \frac{y^2 + 14y + 48}{y^2 + 4y - 32}
\]
\[
\frac{y^2 + 4y - 21}{y^2 + 3y - 28} \div \frac{y^2 + 14y + 48}{y^2 + 4y - 32}
\]
\[
\frac{y^2 + 4y - 21}{y^2 + 3y - 28} \div \frac{y^2 + 14y + 48}{y^2 + 4y - 32}
\]
\[
(y + 7)(y - 3) \quad (y + 8)(y - 4) \quad (y + 7)(y - 4) \quad (y + 6)(y + 8)
\]
\[
\frac{y - 3}{y + 6}
\]

9. \[
\frac{6x^4 - 15x^3}{4x^2 - 6x} \div \frac{45x - 18x^2}{2x^2 - x - 3}
\]
\[
\frac{6x^4 - 15x^3}{4x^2 - 6x} \div \frac{45x - 18x^2}{2x^2 - x - 3}
\]
\[
\frac{6x^4 - 15x^3}{4x^2 - 6x} \div \frac{45x - 18x^2}{2x^2 - x - 3}
\]
\[
\frac{3x^3(2x - 5)}{2x(2x - 3)} \div \frac{(2x - 3)(x + 1)}{9x(5 - 2x)}
\]
\[
\frac{-3x^3(5 - 2x)}{2x(2x - 3)} \div \frac{(2x - 3)(x + 1)}{9x(5 - 2x)}
\]
\[
\frac{-x(x + 1)}{6}
\]

10. \[
\frac{3x^2y^2 - 12x^2}{y^2 + 10y + 16} \div \frac{xy^2 - 12xy + 35x}{y^2 + 3y - 40}
\]
\[
\frac{3x^2y^2 - 12x^2}{y^2 + 10y + 16} \div \frac{xy^2 - 12xy + 35x}{y^2 + 3y - 40}
\]
\[
\frac{3x^2y^2 - 12x^2}{y^2 + 10y + 16} \div \frac{xy^2 - 12xy + 35x}{y^2 + 3y - 40}
\]
\[
\frac{3x^2(y^2 - 4)}{(y + 8)(y + 2)} \div \frac{(y + 8)(y - 5)}{x(y^2 - 12y + 35)}
\]
\[
\frac{3x^2(y - 2)(y + 2)}{(y + 8)(y + 2)} \div \frac{(y + 8)(y - 5)}{x(y^2 - 7y - 5)}
\]
\[
\frac{3x(y - 2)}{y - 7}
\]