PARALLEL, PERPENDICULAR, VERTICAL AND HORIZONTAL LINES

Definitions:

- **Parallel Lines:** are two lines in the same plane that never intersect.
- **Perpendicular Lines:** are two lines that intersect to form a 90° angle.
- **Vertical Lines:** always have the equation $x = c$, for some constant c. For example, the equation of the vertical line through (a, b) is $x = a$.
- **Horizontal Lines:** always have the equation $y = c$, for some constant c. For example, the equation of the horizontal line through (a, b) is $y = b$.

Important Properties:

- Two lines are parallel if and only if they have exactly the same slope.
- Two lines are perpendicular if and only if the product of their slopes is -1. In other words, their slopes are negative reciprocals of one another.
- Vertical lines have undefined slope (or no slope).
- Horizontal lines have zero slope.
- A vertical line is parallel to another vertical line. Similarly, a horizontal line is parallel to another horizontal line.
- A vertical line is perpendicular to a horizontal line. Similarly, a horizontal line is perpendicular to a vertical line.

Common Mistakes to Avoid:

- Be sure to reduce all fractions into lowest terms. For example, if line 1 has slope $\frac{2}{3}$ and line 2 has slope $\frac{4}{6}$, then these lines are parallel since $\frac{2}{3} = \frac{4}{6}$.
- The product of two slopes must be *negative* one in order for the lines to be perpendicular. For examples, if line 1 has slope $\frac{2}{3}$ and line 2 has slope $\frac{3}{2}$, then these lines are NOT perpendicular since their product is *positive* one.
- Notice that no slope and zero slope are different. A vertical line has no slope while a horizontal line has zero slope.

PROBLEMS

1. Find the equation of the vertical line through $(-6, 3)$.

$$\boxed{x = -6}$$

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2. Find the equation of the horizontal line through $\left(\frac{2}{7}, 4\right)$.

$$\boxed{y = 4}$$

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3. Find the equation of the line parallel to $x = 3$ and passes through $(-9, 5)$

   RECALL: $x = 3$ is a vertical line. One vertical line is parallel to another vertical line. Therefore, the line that we are looking for is the vertical line through $(-9, 5)$.

$$\boxed{x = -9}$$

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4. Find the equation of the line perpendicular to $y = -2$ through $(1, 5)$.

   RECALL: $y = -2$ is a horizontal line and a vertical line is perpendicular to a horizontal line. Therefore, we are looking for the vertical line through $(1, 5)$.

$$\boxed{x = 1}$$

5. Determine whether the following lines are parallel, perpendicular, or neither.

$$3x - 5y = 10 \quad \text{and} \quad 5x + 3y = 7$$

   NOTE: First, put each line in slope-intercept form. Once in this form, the slope is the coefficient on x.

$$\begin{aligned} 3x - 5y &= 10 \\ -5y &= -3x + 10 \\ y &= \frac{3}{5}x - 2 \end{aligned} \qquad \begin{aligned} 5x + 3y &= 7 \\ 3y &= -5x + 7 \\ y &= -\frac{5}{3}x + \frac{7}{3} \end{aligned}$$

$$\text{slope} = \frac{3}{5} \qquad \text{slope} = -\frac{5}{3}$$

$$\text{Since } \frac{3}{5} \cdot \left(-\frac{5}{3}\right) = -1,$$

$$\boxed{\text{the two lines are perpendicular}}$$

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6. Determine whether the following lines are parallel, perpendicular, or neither.

$$2x + 5y = -9 \quad \text{and} \quad 6x + 15y = 3$$

$$\begin{aligned} 2x + 5y &= -9 \\ 5y &= -2x - 9 \\ y &= -\frac{2}{5}x - \frac{9}{5} \end{aligned} \qquad \begin{aligned} 6x + 15y &= 3 \\ 15y &= -6x + 3 \\ y &= -\frac{6}{15}x + \frac{3}{15} \\ y &= -\frac{2}{5}x + \frac{1}{5} \end{aligned}$$

$$\text{slope} = -\frac{2}{5} \qquad \text{slope} = -\frac{2}{5}$$

Since the slopes are identical,

$$\boxed{\text{the two lines are parallel.}}$$

7. Find the equation of the line parallel to the $2x - 3y = 10$, passing through $(-8, 3)$.

   NOTE: First you need to find the slope of the line $2x - 3y = 10$ by placing it in slope-intercept form.

$$\begin{aligned} 2x - 3y &= 10 \\ -3y &= -2x + 10 \\ y &= \frac{2}{3}x - \frac{10}{3} \end{aligned}$$

Therefore, $m = \frac{2}{3}$

$$\begin{aligned} y - 3 &= \frac{2}{3}(x + 8) \\ y - 3 &= \frac{2}{3}x + \frac{16}{3} \end{aligned}$$

$$\boxed{y = \frac{2}{3}x + \frac{25}{3}}$$

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8. Find the equation of the line parallel to $3x = 4y + 5$ through $(2, -3)$

   NOTE: Once again, you need to find the slope of $3x = 4y + 5$ by placing it in slope-intercept form.

$$\begin{aligned} 3x &= 4y + 5 \\ 3x - 5 &= 4y \\ \frac{3}{4}x - \frac{5}{4} &= y \end{aligned}$$

Therefore, $m = \frac{3}{4}$

$$\begin{aligned} y + 3 &= \frac{3}{4}(x - 2) \\ y + 3 &= \frac{3}{4}x - \frac{3}{2} \end{aligned}$$

$$\boxed{y = \frac{3}{4}x - \frac{9}{2}}$$

9. Find the equation of the line perpendicular to $2x + 3y = 8$ passing through $(-1, 4)$.

   NOTE: First, we need to find the slope of the given line by placing it in slope-intercept form.

$$\begin{aligned} 2x + 3y &= 8 \\ 3y &= -2x + 8 \\ y &= -\frac{2}{3} + \frac{8}{3} \end{aligned}$$

Therefore, $m = -\frac{2}{3}$

Slope of perpendicular line $= \frac{3}{2}$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= \frac{3}{2}(x - (-1)) \\ y - 4 &= \frac{3}{2}(x + 1) \\ y - 4 &= \frac{3}{2}x + \frac{3}{2} \\ y &= \frac{3}{2}x + \frac{11}{2} \end{aligned}$$

$$\boxed{y = \frac{3}{2}x + \frac{11}{2}}$$

10. Find the equation of the line perpendicular to $2x - y = 7$ passing through $(8, 5)$.

    NOTE: Find the slope of the given line by placing it in slope-intercept form.

$$\begin{aligned} 2x - y &= 7 \\ -y &= -2x + 7 \\ y &= 2x - 7 \end{aligned}$$

Therefore, $m = 2$

$$\text{Slope of perpendicular line } = -\frac{1}{2}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= -\frac{1}{2}(x - 8) \\ y - 5 &= -\frac{1}{2}x + 4 \\ y &= -\frac{1}{2}x + 9 \end{aligned}$$

$$\boxed{y = -\frac{1}{2}x + 9}$$

11. Find the equation of the line passing through $(-1, 4)$ which is perpendicular to the line passing through $(2, 3)$ and $(4, 2)$.

    NOTE: First, we must find the slope of the line passing through $(2, 3)$ and $(4, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{2 - 4} = \frac{1}{-2} = -\frac{1}{2}.$$

Slope of the perpendicular line $= 2$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 2(x - (-1)) \\ y - 4 &= 2(x + 1) \\ y - 4 &= 2x + 2 \\ y &= 2x + 6 \end{aligned}$$

$$\boxed{y = 2x + 6}$$

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12. Find the equation of the line parallel to $6x + 2y = 7$ passing through $(0, 4)$.

    NOTE: $(0, 4)$ is the $y-$intercept of the line. Therefore, we can use the slope-intercept form once we have the slope.

$$\begin{aligned} 6x + 2y &= 7 \\ 2y &= -6x + 7 \\ y &= -3x + \frac{7}{2} \end{aligned}$$

Therefore, $m = -3$

Slope of parallel line $= -3$.

$$\begin{aligned} y &= mx + b \\ y &= -3x + 4 \end{aligned}$$

$$\boxed{y = -3x + 4}$$