Definitions:

- **Quadratic function**: is a function that can be written in the form
  \[ f(x) = ax^2 + bx + c \]
  where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

- **Parabola**: The graph of a squaring function is called a parabola. It is a U-shaped graph.

- **Vertex of a parabola**: The point on the parabola where the graph changes direction. It is the lowest point if \( a > 0 \), and it is the highest point if \( a < 0 \).

- **Standard form of a quadratic function**: A quadratic function \( f(x) = ax^2 + bx + c \) can be expressed in the standard form
  \[ f(x) = a(x - h)^2 + k \]
  by completing the square.

Important Properties:

- **Extreme values of a quadratic function**: Consider the quadratic function \( f(x) = a(x - h)^2 + k \).
  - If \( a > 0 \), then the parabola opens up. Therefore, the **minimum value** of \( f \) occurs at \( x = h \) and its value is \( f(h) = k \).
  - If \( a < 0 \), the parabola opens down. Therefore, the **maximum value** of \( f \) occurs at \( x = h \) and its value is \( f(h) = k \).

- **Vertex Formula**: Given the quadratic \( f(x) = ax^2 + bx + c \), the vertex is found using
  \[ \left( -\frac{b}{2a}, f\left( -\frac{b}{2a} \right) \right) \].

Common Mistakes to Avoid:

- Notice that the maximum or minimum value is the \( y \)-coordinate of the parabola’s vertex. Do not record the extreme value as the \( x \)-coordinate.

- Determining whether the \( y \)-coordinate of the vertex is a maximum or minimum depends on whether \( a \) is positive or negative. It does NOT depend on whether the \( y \)-coordinate is positive or negative.
1. Find the maximum or minimum value of each quadratic function. State whether it is a maximum or minimum.

(a) \( f(x) = 3x^2 - 6x + 2 \)

Here, we know that \( a = 3 \), \( b = -6 \) and \( c = 2 \). Since \( a > 0 \), we know that the \( y \)-coordinate of the vertex is a minimum. However, to find the \( y \)-coordinate of our vertex we first need to find the \( x \)-coordinate of the vertex by using \( x = -\frac{b}{2a} \).

\[
x = -\frac{b}{2a} = -\frac{-6}{2(3)} = \frac{6}{6} = 1.
\]

Now that we have the \( x \)-coordinate, we can find the \( y \)-coordinate of the vertex by finding \( f(1) \).

\[
f(1) = 3(1)^2 - 6(1) + 2 = 3 - 6 + 2 = -1.
\]

Minimum = \(-1\)

(b) \( f(x) = -2x^2 + 8x + 9 \)

Here, we know that \( a = -2 \), \( b = 8 \) and \( c = 9 \). Since \( a < 0 \), we know that the \( y \)-coordinate of the vertex is a maximum. However, to find the \( y \)-coordinate of our vertex we first need to find the \( x \)-coordinate of the vertex by using \( x = -\frac{b}{2a} \).

\[
x = -\frac{b}{2a} = -\frac{8}{2(-2)} = -\frac{8}{-4} = 2
\]

Now that we have the \( x \)-coordinate, we can find the \( y \)-coordinate of the vertex by finding \( f(2) \).

\[
f(2) = -2(2)^2 + 8(2) + 9 = -2(4) + 16 + 9 = -8 + 16 + 9 = 17.
\]

Maximum = \( 17 \)
(c) \( f(x) = 2x^2 - 3x + 1 \)

Here, we know that \( a = 2 \), \( b = -3 \) and \( c = 1 \). Since \( a > 0 \), we know that the \( y \)-coordinate of the vertex is a minimum. However, to find the \( y \)-coordinate of our vertex we first need to find the \( x \)-coordinate of the vertex by using \( x = -\frac{b}{2a} \).

\[
x = -\frac{b}{2a} \\
= -\frac{-3}{2(2)} \\
= -\frac{-3}{4} \\
= \frac{3}{4}
\]

Now that we have the \( x \)-coordinate, we can find the \( y \)-coordinate of the vertex by finding \( f \left( \frac{3}{4} \right) \).

\[
f \left( \frac{3}{4} \right) = 2 \left( \frac{3}{4} \right)^2 - 3 \left( \frac{3}{4} \right) + 1 \\
= 2 \left( \frac{9}{16} \right) - \frac{9}{4} + 1 \\
= \frac{9}{8} - \frac{9}{4} + 1 \\
= \frac{9}{8} - \frac{18}{8} + \frac{8}{8} \\
= \frac{-1}{8}
\]

\[\text{Minimum} = -\frac{1}{8}\]

(d) \( f(x) = -x^2 - 8x + 7 \)

Here, we have \( a = -1 \), \( b = -8 \) and \( c = 7 \). Since \( a < 0 \), we know that the \( y \)-coordinate of the vertex is a maximum. However, to find the \( y \)-coordinate of our vertex we first need to find the \( x \)-coordinate of the vertex by using \( x = -\frac{b}{2a} \).

\[
x = -\frac{b}{2a} \\
= -\frac{-8}{2(-1)} \\
= -\frac{-8}{-2} \\
= 4
\]

Now that we have the \( x \)-coordinate, we can find the \( y \)-coordinate of the vertex by finding \( f(-4) \).

\[
f(-4) = -(-4)^2 - 8(-4) + 7 \\
= -16 + 32 + 7 \\
= 23
\]

\[\text{Maximum} = 23\]

NOTE: you can also solve the above problems by placing the quadratic in standard form and identifying the \( y \)-coordinate that way.
2. During the annual frog jumping contest at the county fair, the height of the frog’s jump, in feet, is given by

\[ f(x) = -\frac{1}{3}x^2 + \frac{4}{3}x. \]

What was the maximum height reached by the frog?

Since \( f \) is a quadratic with \( a = -\frac{1}{3} < 0 \), the maximum height of the frog’s jump is found by identifying the \( y \)-coordinate of the parabola’s vertex. To do this note that \( a = -\frac{1}{3}, \ b = \frac{4}{3} \) and \( c = 0 \). First, we find the \( x \)-coordinate by using \( -\frac{b}{2a} \).

\[
x = -\frac{b}{2a} = -\frac{4/3}{2(-1/3)} = -\frac{4/3}{-2/3} = 2.
\]

Next, we will have the maximum height once we find \( f(2) \).

\[
f(2) = -\frac{1}{3}(2)^2 + \frac{4}{3}(2) = -\frac{1}{3}(4) + \frac{8}{3} = \frac{4}{3} + \frac{8}{3} = \frac{4}{3}.
\]

Maximum height of frog’s jump = \( \frac{4}{3} \) ft

3. A ball is thrown directly upward from an initial height of 50 feet. If the height, in feet, of the ball after \( t \) seconds is given by

\[ f(t) = -16t^2 + 40t + 50, \]

find the maximum height reached by the ball.

Since \( f \) is a quadratic with \( a = -16 < 0 \), the \( f(t) \) coordinate of the parabola’s vertex will identify the maximum height. To find the \( t \)-coordinate of the vertex, we use \( t = -\frac{b}{2a} \) where \( a = -16 \) and \( b = 40 \).

\[
t = -\frac{b}{2a} = -\frac{40}{2(-16)} = -\frac{40}{-32} = \frac{5}{4}.
\]

To find the maximum, we need to evaluate \( f\left(\frac{5}{4}\right) \).

\[
f\left(\frac{5}{4}\right) = -16\left(\frac{5}{4}\right)^2 + 40\left(\frac{5}{4}\right) + 50 = -16\left(\frac{25}{16}\right) + 50 + 50 = -25 + 100 = 75.
\]

Maximum height of ball = 75 ft
4. A toy rocket is launched from the top of a 150 foot cliff. If the height, in feet, of the rocket \( t \) seconds after liftoff is given by

\[ f(t) = -16t^2 + 288t + 150, \]

find the maximum height of the rocket and the time it reaches its maximum height.

Since \( f \) is a quadratic with \( a = -16 < 0 \), the \( t \)–coordinate of the parabola’s vertex will identify the time the rocket reaches its maximum height and the \( f(t) \) coordinate of the vertex will give the maximum height. We first find the time \( t \) by using \( t = -\frac{b}{2a} \) where \( a = -16 \) and \( b = 288 \).

\[
t = -\frac{b}{2a} = -\frac{288}{2(-16)} = \frac{288}{32} = 9.
\]

Next, to find the maximum height, we evaluate \( f(9) \).

\[
f(9) = -16(9)^2 + 288(9) + 150 = -16(81) + 2592 + 150 = -1296 + 2592 + 150 = 1446.
\]

\[
\text{Time to reach max height} = 9 \text{ seconds}
\]

\[
\text{Maximum height of rocket} = 1446 \text{ ft}
\]