Definition:

- **Radical equation**: is an equation with one or more radical expressions.

Important Properties:

- If both sides of an equation are raised to the same power, all solutions of the original equation are also solutions of the new equation.
- When solving an equation involving a even indexed radical, you must always check your "answers" in the *original* equation. We may end up with one or more extraneous solutions. Extraneous solutions may be introduced when we raise both sides of an equation to an even power.

Solving an equation involving a radical:

1. Isolate the radical. Make sure that one radical term is alone on one side of the equation.
2. Raise both *sides* of the equation to the same power that is the same as the index on the radical.
3. Solve the resulting equation. If it still contains a radical, repeat steps 1 and 2.
4. Check your possible solutions in the original equation. Exclude any that result in a false statement.

Common Mistakes to Avoid:

- Remember to raise both *sides* of the equation to the same power. Do NOT raise each *term* to the same power. For example, if $a+b=c$ then raising both sides to the same power we get $(a+b)^n = c^n$. We do NOT get $a^n + b^n = c^n$.
- Remember that $(x+y)^2 \neq x^2+y^2$. Instead, $(x+y)^2 = x^2+2xy+y^2$. Do not forget the middle term. (Note, this formula arises from foil).
- Do NOT forget to check your answers in the original equation.

PROBLEMS

Solve for x in each of the following equations.

1. $\sqrt{x-3} = 4$

Since the square root is already isolated, we start by squaring both sides and solving.

$$\begin{aligned} \sqrt{x-3} &= 4 \\ \left(\sqrt{x-3}\right)^2 &= 4^2 \\ x-3 &= 16 \\ x &= 19 \end{aligned}$$

We need to check our possible solution in the original equation.

Check: $x = 19$

$$\begin{aligned} \sqrt{19-3} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4\star \end{aligned}$$

$$\boxed{x = 4}$$

2. $\sqrt{3x-6} - 2 = 1$

Before we square both sides, we need to isolate the radical on one side of the equation.

$$\begin{aligned} \sqrt{3x-6} - 2 &= 1 \\ \sqrt{3x-6} &= 3 \\ \left(\sqrt{3x-6}\right)^2 &= 3^2 \\ 3x-6 &= 9 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Check: $x = 5$

$$\begin{aligned} \sqrt{3(5)-6} - 2 &= 1 \\ \sqrt{15-6} - 2 &= 1 \\ \sqrt{9} - 2 &= 1 \\ 3 - 2 &= 1 \\ 1 &= 1\star \end{aligned}$$

$$\boxed{x = 5}$$

3. $x + \sqrt{x-4} = 4$

We must isolate the radical before squaring both sides.

$$\begin{aligned} x + \sqrt{x-4} &= 4 \\ \sqrt{x-4} &= 4 - x \\ \left(\sqrt{x-4}\right)^2 &= (4-x)^2 \\ x - 4 &= 16 - 8x + x^2 \\ x &= x^2 - 8x + 20 \\ 0 &= x^2 - 9x + 20 \\ 0 &= (x-4)(x-5) \end{aligned}$$

$$\begin{aligned} x - 4 &= 0 \\ x &= 4 \end{aligned} \qquad \begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

Next, we must check each possible solution in the original equation.

Check: $x = 4$

$$\begin{aligned} 4 + \sqrt{4-4} &= 4 \\ 4 + 0 &= 4 \\ 4 &= 4\star \end{aligned}$$

Check: $x = 5$

$$\begin{aligned} 5 + \sqrt{5-4} &= 4 \\ 5 + 1 &= 4 \\ 6 &\neq 4 \end{aligned}$$

$\boxed{x = 4}$

4. $\sqrt{x+4} - \sqrt{x-4} = 2$

We must isolate *one* of the radicals before we can square both sides.

$$\begin{aligned} \sqrt{x+4} - \sqrt{x-4} &= 2 \\ \sqrt{x+4} &= \sqrt{x-4} + 2 \\ \left(\sqrt{x+4}\right)^2 &= \left(\sqrt{x-4} + 2\right)^2 \\ x + 4 &= x - 4 + 4\sqrt{x-4} + 4 \\ x + 4 &= x + 4\sqrt{x-4} \\ 4 &= 4\sqrt{x-4} \\ (4)^2 &= \left(4\sqrt{x-4}\right)^2 \\ 16 &= 16(x-4) \\ 16 &= 16x - 64 \\ 80 &= 16x \\ 5 &= x \end{aligned}$$

Check: $x = 5$

$$\begin{aligned} \sqrt{5+4} - \sqrt{5-4} &= 2 \\ \sqrt{9} - \sqrt{1} &= 2 \\ 3 - 1 &= 2 \\ 2 &= 2\star \end{aligned}$$

$\boxed{x = 5}$

5. $\sqrt[3]{x-2}+4=2$

We must isolate the radical before we can raise both sides of the equation to the third power.

$$\begin{aligned}\sqrt[3]{x-2}+4&=2\\ \sqrt[3]{x-2}&=-2\\ \left(\sqrt[3]{x-2}\right)^3&=(-2)^3\\ x-2&=-8\\ x&=-6\end{aligned}$$

$$\boxed{x=-6}$$

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6. $\sqrt{2x+3}-\sqrt{x-2}=2$

We first need to isolate one of the radicals on one side.

$$\begin{aligned}\sqrt{2x+3}-\sqrt{x-2}&=2\\ \sqrt{2x+3}&=\sqrt{x-2}+2\\ \left(\sqrt{2x+3}\right)^2&=\left(\sqrt{x-2}+2\right)^2\\ 2x+3&=x-2+4\sqrt{x-2}+4\\ 2x+3&=x+2+4\sqrt{x-2}\\ x+1&=4\sqrt{x-2}\\ (x+1)^2&=\left(4\sqrt{x-2}\right)^2\\ x^2+2x+1&=16(x-2)\\ x^2+2x+1&=16x-32\\ x^2-14x+33&=0\\ (x-11)(x-3)&=0\end{aligned}$$

$x-11=0$	$x-3=0$
$x=11$	$x=3$

We now need to check our possible solutions into the original equation.

Check: $x=11$

$$\begin{aligned}\sqrt{2(11)+3}-\sqrt{(11)-2}&=2\\ \sqrt{22+3}-\sqrt{9}&=2\\ \sqrt{25}-\sqrt{9}&=2\\ 5-3&=2\\ 2&=2\star\end{aligned}$$

Check: $x=3$

$$\begin{aligned}\sqrt{2(3)+3}-\sqrt{3-2}&=2\\ \sqrt{6+3}-\sqrt{1}&=2\\ \sqrt{9}-1&=2\\ 3-1&=2\\ 2&=2\star\end{aligned}$$

$$\boxed{x=11,\quad x=3}$$

7. $\sqrt{2x+5}+\sqrt{x+2}=5$

$$\begin{aligned}
\sqrt{2x+5}+\sqrt{x+2}&=5\\
\sqrt{2x+5}&=5-\sqrt{x+2}\\
\left(\sqrt{2x+5}\right)^2&=\left(5-\sqrt{x+2}\right)^2\\
2x+5&=25-10\sqrt{x+2}+x+2\\
2x+5&=27+x-10\sqrt{x+2}\\
x-22&=-10\sqrt{x+2}\\
(x-22)^2&=\left(-10\sqrt{x+2}\right)^2\\
x^2-44x+484&=100(x+2)\\
x^2-44x+484&=100x+200\\
x^2-144x+284&=0\\
(x-2)(x-142)&=0
\end{aligned}$$

$$\begin{array}{l|l}
x-2=0 & x-142=0\\
x=2 & x=142
\end{array}$$

We now need to check our possible solutions into the original equation.

Check: $x=2$

$$\begin{aligned}
\sqrt{2(2)+5}+\sqrt{2+2}&=5\\
\sqrt{9}+\sqrt{4}&=5\\
3+2&=5\\
5&=5\star
\end{aligned}$$

Check: $x=142$

$$\begin{aligned}
\sqrt{2(142)+5}+\sqrt{142+2}&=5\\
\sqrt{289}+\sqrt{144}&=5\\
17+12&=5\\
29&\neq 5
\end{aligned}$$

$$\boxed{x=2}$$

8. $\sqrt{x+3}+\sqrt{2-x}=3$

$$\begin{aligned}
\sqrt{x+3}+\sqrt{2-x} &= 3 \\
\sqrt{x+3} &= 3-\sqrt{2-x} \\
\left(\sqrt{x+3}\right)^2 &= \left(3-\sqrt{2-x}\right)^2 \\
x+3 &= 9-6\sqrt{2-x}+2-x \\
x+3 &= 11-x-6\sqrt{2-x} \\
2x-8 &= -6\sqrt{2-x} \\
(2x-8)^2 &= \left(-6\sqrt{2-x}\right)^2 \\
4x^2-32x+64 &= 36(2-x) \\
4x^2-32x+64 &= 72-36x \\
4x^2+4x-8 &= 0 \\
4(x^2+x-2) &= 0 \\
4(x+2)(x-1) &= 0
\end{aligned}$$

$$\begin{aligned}
x+2 &= 0 \\
x &= -2
\end{aligned}$$

$$\begin{aligned}
x-1 &= 0 \\
x &= 1
\end{aligned}$$

We now need to check our possible solutions into the original equation.

Check: $x=-2$

$$\begin{aligned}
\sqrt{-2+3}+\sqrt{2-(-2)} &= 3 \\
\sqrt{1}+\sqrt{4} &= 3 \\
1+2 &= 3 \\
3 &= 3\star
\end{aligned}$$

Check: $x=1$

$$\begin{aligned}
\sqrt{1+3}+\sqrt{2-1} &= 3 \\
\sqrt{4}+\sqrt{1} &= 3 \\
2=1 &= 3 \\
3 &= 3\star
\end{aligned}$$

$$\boxed{x=2, \quad x=1}$$