Important Properties:

- **Difference of squares:**
  \[ a^2 - b^2 = (a - b)(a + b) \]

- **Difference of cubes:**
  \[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

- **Sum of cubes:**
  \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]

- Remember that you can always check your factoring by multiplication.
- Always look for the greatest common factor (GCF) first.
- Remember that an expression containing four terms will most likely factor by grouping.

Common Mistakes to Avoid:

- Remember to factor out the variable to its *smallest* exponent.
- Do NOT forget to carry your GCF all the way to the end of the problem.

**PROBLEMS**

Factor completely.

1. \[ x^{1/2} - x^{5/2} \]

   \[
   \begin{align*}
   x^{1/2} - x^{5/2} & \\
   x^{1/2} (1 - x^2) & \\
   x^{1/2}(1 - x)(1 + x) & 
   \end{align*}
   \]

2. \[ x^{-1/2}(x + 3)^{1/2} + x^{1/2}(x + 3)^{-1/2} \]

   \[
   \begin{align*}
   x^{-1/2}(x + 3)^{1/2} + x^{1/2}(x + 3)^{-1/2} & \\
   x^{-1/2}(x + 3)^{-1/2} [(x + 3) + x] & \\
   x^{-1/2}(x + 3)^{-1/2}(2x + 3) & 
   \end{align*}
   \]
3. \( x^{3/2} + 8x^{1/2} + 15x^{-1/2} \)
\[
x^{3/2} + 8x^{1/2} + 15x^{-1/2} = x^{-1/2}(x^2 + 8x + 15)
\]
\[
= x^{-1/2}(x + 3)(x + 5)
\]

4. \( 2x^{1/2} + 5x^{-1/2} + 2x^{-3/2} \)
\[
2x^{1/2} + 5x^{-1/2} + 2x^{-3/2} = x^{-3/2}(2x^2 + 5x + 2)
\]
\[
= x^{-3/2}(2x + 1)(x + 2)
\]

5. \( (x + 2)^{7/2} - (x + 2)^{3/2} \)
\[
(x + 2)^{7/2} - (x + 2)^{3/2} = (x + 2)^{3/2} [(x + 2)^2 - 1]
\]
\[
= (x + 2)^{3/2}(x^2 + 4x + 4 - 1)
\]
\[
= (x + 2)^2(x^2 + 4x + 3)
\]
\[
= (x + 2)^2(x + 1)(x + 3)
\]

OR
\[
(x + 2)^{7/2} - (x + 2)^{3/2} = (x + 2)^{3/2} [(x + 2)^2 - 1]
\]
\[
= (x + 2)^2[(x + 2) - 1][(x + 2) + 1]
\]
\[
= (x + 2)^2(x + 2 - 1)(x + 2 + 1)
\]
\[
= (x + 2)^2(x + 1)(x + 3)
\]

6. \( 4(x + 1)^{1/2} - 5(x + 1)^{3/2} + (x + 1)^{5/2} \)
\[
4(x + 1)^{1/2} - 5(x + 1)^{3/2} + (x + 1)^{5/2} = (x + 1)^{1/2} [4 - 5x + 5x^2 + 2x + 1]
\]
\[
= (x + 1)^{1/2}(x^2 - 3x)
\]
\[
= x(x + 1)^{1/2}(x - 3)
\]

OR
\[
4(x + 1)^{1/2} - 5(x + 1)^{3/2} + (x + 1)^{5/2} = (x + 1)^{1/2} [4 - 5x + 5 + x^2 + 2x + 1]
\]
\[
= (x + 1)^{1/2}(x^2 + 4x + 3)
\]
\[
= (x + 1)^{1/2}(x + 1 - 1)(x + 1 - 4)
\]
\[
= x(x + 1)^{1/2}(x - 3)
\]

7. \( x^{1/2} - 3x^{1/3} - 3x^{1/6} - 9 \)
\[
x^{1/2} - 3x^{1/3} - 3x^{1/6} - 9 = x^{1/3}(x^{1/6} - 3) - 3(x^{1/6} - 3)
\]
\[
= (x^{1/6} - 3)(x^{1/3} - 3)
\]
8. \(12x^{4/3} - 5x^{2/3} - 2\)

\[
\frac{12x^{4/3} - 5x^{2/3} - 2}{(3x^{2/3} - 2)(4x^{2/3} + 1)}
\]

OR

Letting \(u = x^{2/3}\)

\[
12u^2 - 5u - 2
\]

\[
(3u - 2)(4u + 1)
\]

\[
(3x^{2/3} - 2)(4x^{2/3} + 1)
\]

9. \(x^{3/5} + 5x^{2/5} - 3x^{1/5} - 15\)

\[
\frac{x^{3/5} + 5x^{2/5} - 3x^{1/5} - 15}{x^{2/5}(x^{1/5} + 5) - 3(x^{1/5} + 5)}
\]

\[
(x^{1/5} + 5)(x^{2/5} - 3)
\]