Definitions:

- Slope: of a line tells how fast y changes for each unit of change in x.
- Linear equation in two variables: is an equation that can be written as

$$ax + by = c$$

where a, b, and c are real numbers and a and b cannot both be zero.

Important Formulas:

• Slope formula: The slope of the line through the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

Note that it does not matter if you start with y_1 or y_2 . However, you must start with its corresponding x in the denominator.

• Slope-intercept form: The slope-intercept form of an equation with slope m and y-intercept b is given by

$$y = mx + b.$$

• Point-slope formula: The equation of the line with slope m and passing through (x_1, y_1) can be found using

$$y - y_1 = m(x - x_1).$$

Common Mistakes to Avoid:

- When identifying the slope and y-intercept using the slope-intercept form, remember to divide each term by the coefficient on y. The slope and y-intercept can only be identified once you have isolated y.
- \bullet Remember that the change in y is in the numerator of the slope formula. DO NOT place it in the denominator.

PROBLEMS

1. Identify the slope and the y-intercept of each line.

(a)
$$3x - 2y = 6$$

$$3x - 2y = 6$$
$$-2y = -3x + 6$$
$$y = \frac{3}{2}x - 3$$

$$m = \frac{3}{2}$$

$$y - \text{intercept} = (0, -3)$$

(b) 5x + 10y = -3

$$5x + 10y = -3$$

$$10y = -5x - 3$$

$$y = -\frac{5}{10}x - \frac{3}{10}$$

$$y = -\frac{1}{2}x - \frac{3}{10}$$

$$\boxed{m = -\frac{1}{2}}$$

$$y - \text{intercept} = \left(0, -\frac{3}{10}\right)$$

2. Find the slope of the line passing through (-1,3) and (5,-2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2 - 3}{5 - (-1)}$$

$$m = -\frac{5}{6}$$

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3. Find the slope of the line passing through (-9,2) and (-5,5).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 2}{-5 - (-9)}$$

$$m = \frac{3}{4}$$

$$m = \frac{3}{4}$$

4. Find the equation of the line with slope m = -3 and passes through (5, -2).

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -3(x - 5)$$
$$y + 2 = -3x + 15$$
$$y = -3x + 13$$
$$y = -3x + 13$$

5. Find the equation of the line with $m=\frac{3}{4}$ and passing through (-1,2).

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{3}{4}(x - (-1))$$
$$y - 2 = \frac{3}{4}x + \frac{3}{4}$$
$$y = \frac{3}{4}x + \frac{11}{4}$$

$$y = \frac{3}{4}x + \frac{11}{4}$$

6. Find the equation of the line passing through (-2,3) and (4,-5).

NOTE: First, we must find the slope of the line.

$$m = \frac{-5 - 3}{4 - (-2)} = \frac{-8}{6} = \frac{-4}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{4}{3}(x - (-2))$$

$$y - 3 = -\frac{4}{3}x - \frac{8}{3}$$

$$y = \frac{4}{3} + \frac{1}{3}$$

$$y = \frac{4}{3}x + \frac{1}{3}$$

7. Find the equation of the line passing through (-7,2) and has a y-intercept at 3.

NOTE: First, we must find the slope of the line. Remember that a y-intercept at 3 translates to the ordered pair (0,3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{-7 - 0} = \frac{-1}{-7} = \frac{1}{7}$$
$$y = mx + b$$
$$y = \frac{1}{7}x + 3$$
$$y = \frac{1}{7}x + 3$$

8. Find the equation of the line which has an x-intercept at -2 and a y-intercept at 4.

NOTE: This means that the line passes through (-2,0) and (0,4).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2$$

$$y = mx + b$$

$$y = 2x + 4$$

$$y = 2x + 4$$

9. Find the equation of the line passing through (-7,2) and has an x-intercept at 3.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-7 - 3} = \frac{2}{-10} = -\frac{1}{5}$$
$$y - 0 = -\frac{1}{5}(x - 3)$$
$$y = -\frac{1}{5}x + \frac{3}{5}$$
$$y = -\frac{1}{5}x + \frac{3}{5}$$