Definition:

- **Quadratic Equation**: is an equation that can be written in the form
  
  \[ ax^2 + bx + c = 0, \]

  where \( a, b, \) and \( c \) are real numbers, \( a \neq 0 \).

Important Properties:

- **Quadratic Formula**: The solutions of \( ax^2 + bx + c = 0 \) where \( a \neq 0 \) are given by
  
  \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \]

- The quadratic formula is a result of solving \( ax^2 + bx + c = 0 \) by completing the square.
- The quadratic formula can be used to solve any quadratic equation.
- If \( b^2 - 4ac < 0 \) then there are no real solutions to the quadratic equation.
- If \( b^2 - 4ac = 0 \), then the quadratic equation has only one real zero.
- If \( b^2 - 4ac > 0 \), then the quadratic equation has two real solutions.

Common Mistakes to Avoid:

- Before identifying \( a, b, \) and \( c \) to be used in the quadratic formula, make sure one side of your equation is zero.
- In the quadratic formula, the \(-b\) is also divided by \( 2a \).
- \( \sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \).
- Be careful when simplifying your final answer. Remember that
  \[ \frac{b+ac}{c} \neq b + a, \quad \text{and} \quad \frac{c+a}{c} \neq 1 + a. \]
- The quadratic formula can only be used on a quadratic equation. Do not use on a quadratic-type equation without a change of variables.
Solving quadratic equations by quadratic formula, page 2

PROBLEMS

Solve the following equations using the quadratic formula.

1. \(6x^2 - 5x - 4 = 0\)

   Note, \(a = 6, \ b = -5,\) and \(c = -4.\)

   \[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]

   \[
x = \frac{5 \pm \sqrt{25 + 96}}{12}
   \]

   \[
x = \frac{5 \pm \sqrt{121}}{12}
   \]

   \[
x = \frac{5 \pm 11}{12}
   \]

   \[
x = \frac{5 + 11}{12} = \frac{16}{12} = \frac{4}{3},
   \]

   \[
x = \frac{5 - 11}{12} = \frac{-6}{12} = \frac{-1}{2}
   \]

   \[
x = \frac{4}{3}, \ x = -\frac{1}{2}
   \]

2. \(x^2 + 3x - 2 = 0\)

   Note, \(a = 1, \ b = 3,\) and \(c = -2.\)

   \[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]

   \[
x = \frac{-3 \pm \sqrt{9 + 8}}{2}
   \]

   \[
x = \frac{-3 \pm \sqrt{17}}{2}
   \]

   \[
x = \frac{-3 \pm \sqrt{17}}{2}
   \]

3. \(x^2 - 4 = 2x\)

   We must make one side zero before we can identify \(a, b,\) and \(c.\)

   \[
x^2 - 4 = 2x
   \]

   \[
x^2 - 2x - 4 = 0
   \]

   Therefore, \(a = 1, \ b = -2,\) and \(c = -4.\)

   \[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]

   \[
x = \frac{2 \pm \sqrt{4 + 16}}{2}
   \]

   \[
x = \frac{2 \pm 2\sqrt{5}}{2}
   \]

   \[
x = \frac{2(1 \pm \sqrt{5})}{2}
   \]

   \[
x = 1 \pm \sqrt{5}
   \]

   \[
x = 1 \pm \sqrt{5}
   \]
4. $4x^2 + 4x - 1 = 0$

Note, $a = 4$, $b = 4$, and $c = -1$.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(4)(-1)}}{2(4)}
\]

\[
x = \frac{-4 \pm \sqrt{16 + 16}}{8}
\]

\[
x = \frac{-4 \pm \sqrt{32}}{8}
\]

\[
x = \frac{-4 \pm 4\sqrt{2}}{8}
\]

\[
x = \frac{4(-1 \pm \sqrt{2})}{8}
\]

\[
x = \frac{-1 \pm \sqrt{2}}{2}
\]

\[
x = \frac{1 \pm \sqrt{2}}{2}
\]

5. $5x^2 - 3x = 7$

$5x^2 - 3x = 7$

$5x^2 - 3x - 7 = 0$

Therefore, $a = 5$, $b = -3$, and $c = -7$.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-7)}}{2(5)}
\]

\[
x = \frac{3 \pm \sqrt{9 + 140}}{10}
\]

\[
x = \frac{3 \pm \sqrt{149}}{10}
\]

\[
x = \frac{3 \pm \sqrt{149}}{10}
\]

\[
x = \frac{1 \pm \sqrt{2}}{2}
\]

6. $-2x(x + 4) = -6$

$-2x(x + 4) = -6$

$-2x^2 - 8x = -6$

$-2x^2 - 8x + 6 = 0$

Note, that a $-2$ can be divided out of each term. Therefore, before using the quadratic formula, we will divide each term by $-2$.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}
\]

\[
x = \frac{-4 \pm \sqrt{16 + 12}}{2}
\]

\[
x = \frac{-4 \pm \sqrt{28}}{2}
\]

\[
x = \frac{-4 \pm 2\sqrt{7}}{2}
\]

\[
x = \frac{2(-2 \pm \sqrt{7})}{2}
\]

\[
x = -2 \pm \sqrt{7}
\]

\[
x = -2 \pm \sqrt{7}
\]
7. \(7x^2 - 3x = -6\)

We must first make one side zero before identifying \(a\), \(b\), and \(c\).

\[
\begin{align*}
7x^2 - 3x &= -6 \\
7x^2 - 3x + 6 &= 0
\end{align*}
\]

Therefore, \(a = 7\), \(b = -3\), and \(c = 6\).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(7)(6)}}{2(7)}
\]

\[
x = \frac{3 \pm \sqrt{9 - 168}}{14}
\]

\[
x = \frac{3 \pm \sqrt{-159}}{14}
\]

No real solution

8. \(x^2 + 16 = 8x\)

Once again, we must have one side zero before we can identify \(a\), \(b\), or \(c\).

\[
\begin{align*}
x^2 + 16 &= 8x \\
x^2 - 8x + 16 &= 0
\end{align*}
\]

Therefore, \(a = 1\), \(b = -8\), and \(c = 16\).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}
\]

\[
x = \frac{8 \pm \sqrt{64 - 64}}{2}
\]

\[
x = \frac{8}{2}
\]

\[
x = 4
\]

\(x = 4\)