Definition:

- **Linear equation in two variables**: is an equation that can be written as
  \[ ax + by = c \]
  where \( a \), \( b \), and \( c \) are real numbers and \( a \) and \( b \) cannot both be zero.

Three ways to graph a line:

1. **Plot points**: Choose values for \( x \) (or \( y \)) and find ordered pairs. Then plot these ordered pairs and connect them with a straight line.
2. **Using intercepts**: Find the \( x \)–intercept and \( y \)–intercept of the linear equation. Plot these two points and connect them with a straight line.
3. **Using the slope and \( y \)–intercept**: Recall that placing the equation in slope-intercept form of \( y = mx + b \) identifies the slope and the \( y \)–intercept. Plot the \( y \)–intercept first and then use the slope, \( m = \frac{\text{rise}}{\text{run}} \), to find another point on the graph. Connect these two points with a straight line.

Important Properties:

- The graph of a linear equation in two variables will always be a line.
- The advantage of using the slope and a point to graph a line is that you do not need to have the equation of the line in order to graph it. You only need to know the slope and a point on the graph.
- \( x = c \) represents a vertical line at \( c \).
- \( y = c \) represents a horizontal line at \( c \).
- The \( x \)–intercept is found by setting \( y = 0 \) and solving for \( x \). The \( x \)–intercept is represented by the ordered pair \((x, 0)\).
- The \( y \)–intercept is found by setting \( x = 0 \) and solving for \( y \). The \( y \)–intercept is represented by the ordered pair \((0, y)\).
- When rise is positive you go up and when rise is negative you go down.
- When run is positive you go to the right and when run is negative you go to the left.
- Although it is true that two points determine a line, it is better to plot at least three points in order to avoid mistakes.

Common Mistakes to Avoid:

- When your slope is negative, remember to include the negative with either the numerator or the denominator NOT both.
PROBLEMS

1. **Graph** $2x + 3y = 6$.

For this problem we will graph the equation using the $x$– and $y$–intercepts. To find the $x$–intercept we substitute $y = 0$ and find that

$$2x + 3(0) = 6$$

$$2x = 6$$

$$x = 3.$$ 

For the $y$–intercept we substitute $x = 0$ into the equation and find that

$$2(0) + 3y = 6$$

$$3y = 6$$

$$y = 2$$

Now, plotting the $x$–intercept of $(3,0)$ and the $y$–intercept of $(0,2)$ and connecting them with a straight line, we get the following graph of the equation.

2. **Graph** $5x - 2y = 10$.

We will graph this line again by finding the $x$– and $y$–intercepts. To find the $x$–intercept, we let $y = 0$ and find that

$$5x - 2(0) = 10$$

$$5x = 10$$

$$x = 2$$

For the $y$–intercept, we let $x = 0$ and get

$$5(0) - 2y = 10$$

$$-2y = 10$$

$$y = -5$$

Therefore, plotting the intercepts of $(0, -5)$ and $(2,0)$ and connecting them with a straight line, we get the following graph.
3. **Graph** $3x + 2y = 7$.

We will graph this line by plotting points. Choosing $x = 1$, we find

$$3(1) + 2y = 7$$
$$3 + 2y = 7$$
$$2y = 4$$
$$y = 2$$

Choosing $x = -1$, we have

$$3(-1) + 2y = 7$$
$$-3 + 2y = 7$$
$$2y = 10$$
$$y = 5$$

Finally, choosing $x = 3$, we find that

$$3(3) + 2y = 7$$
$$9 + 2y = 7$$
$$2y = -2$$
$$y = -1$$

Therefore, when we graph the points $(1, 2)$, $(-1, 5)$, and $(3, -1)$ and connecting them with a straight line, we obtain the following graph.

![Graph of $3x + 2y = 7$](image)

4. **Graph** $-3x + 4y = 5$.

We will graph this line by plotting points. If we choose $x = -1$, then

$$-3(-1) + 4y = 5$$
$$3 + 4y = 5$$
$$4y = 2$$
$$y = \frac{1}{2}$$

Choosing $x = 1$, we find

$$-3(1) + 4y = 5$$
$$3 + 4y = 5$$
$$4y = 8$$
$$y = 2$$

Finally, choosing $x = -3$, we have

$$-3(-3) + 4y = 5$$
$$9 + 4y = 5$$
$$4y = -4$$
$$y = -1$$

Now, plotting the points $(-1, \frac{1}{2})$, $(1, 2)$, and $(-3, -1)$ and connecting them with a straight line, we obtain the following graph.

![Graph of $-3x + 4y = 5$](image)
5. **Graph** \(2x + 3y = 12\).

We will graph this and the remaining lines using the slope and a point. In order to do this we first need to place the equation in slope-intercept form.

\[
2x + 3y = 12 \\
3y = -2x + 12 \\
y = -\frac{2}{3}x + 4
\]

Therefore, the \(y\)-intercept is \((0, 4)\) and the slope \(m = -\frac{2}{3}\). So, we will plot the point \((0, 4)\) and then rise \(-2\) (go down 2 units) and run 3 (go right 3 units). This gives us our second point on the graph as \((3, 2)\). Plotting these two points and connecting them with a straight line, we obtain the following graph.

6. **Graph the line with** \(m = \frac{2}{5}\) **and which passes through** \((-2, 1)\).

We will use the slope and point given to graph this. First, we will plot the point \((-2, 1)\). Next, we will use the slope of \(m = \frac{2}{5}\) and rise 2 (go up 2 units) and run 5 (go right 5 units). This gives us our second point at \((3, 3)\). Connecting these points we get the following graph.

7. **Graph the line with slope** \(m = -\frac{3}{2}\) **and passes through** \((-3, -2)\).

First, we will plot the point \((-3, -2)\). Then using the slope \(m = -\frac{3}{2} = -\frac{3}{2}\), we will rise \(-3\) (go down 3 units) and run 2 (go right 2 units). This gives us our second point at \((-1, -5)\). Connecting these points we will get the following graph.