Definitions:

- **Slope**: of a line tells how fast $y$ changes for each unit of change in $x$.
- **Linear equation in two variables**: is an equation that can be written as $ax + by = c$ where $a, b,$ and $c$ are real numbers and $a$ and $b$ cannot both be zero.

Important Formulas:

- **Slope formula**: The slope of the line through the points $(x_1, y_1)$ and $(x_2, y_2)$ is given by

  $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

  Note that it does not matter if you start with $y_1$ or $y_2$. However, you must start with its corresponding $x$ in the denominator.

- **Slope-intercept form**: The slope-intercept form of an equation with slope $m$ and $y$–intercept $b$ is given by

  $$y = mx + b.$$ 

- **Point-slope formula**: The equation of the line with slope $m$ and passing through $(x_1, y_1)$ can be found using

  $$y - y_1 = m(x - x_1).$$

Common Mistakes to Avoid:

- When identifying the slope and $y$–intercept using the slope-intercept form, remember to divide each term by the coefficient on $y$. The slope and $y$–intercept can only be identified once you have isolated $y$.
- Remember that the change in $y$ is in the numerator of the slope formula. DO NOT place it in the denominator.
PROBLEMS

1. Identify the slope and the \( y \)-intercept of each line.

   (a) \( 3x - 2y = 6 \)
   
   \[
   \begin{align*}
   3x - 2y &= 6 \\
   -2y &= -3x + 6 \\
   y &= \frac{3}{2}x - 3 \\
   
   \frac{m}{2} &= 3 \\
   
   y - \text{intercept} &= (0, -3)
   \end{align*}
   \]

   (b) \( 5x + 10y = -3 \)
   
   \[
   \begin{align*}
   5x + 10y &= -3 \\
   10y &= -5x - 3 \\
   y &= -\frac{5}{10}x - \frac{3}{10} \\
   y &= -\frac{1}{2}x - \frac{3}{10} \\
   
   \frac{m}{2} &= -\frac{1}{2} \\
   
   y - \text{intercept} &= \left(0, -\frac{3}{10}\right)
   \end{align*}
   \]

2. Find the slope of the line passing through \((-1, 3)\) and \((5, -2)\).

   \[
   m = \frac{y_2 - y_1}{x_2 - x_1} \\
   m = \frac{-2 - 3}{5 - (-1)} \\
   m = \frac{5}{6} \\
   
   \frac{m}{2} = \frac{5}{6}
   \]

3. Find the slope of the line passing through \((-9, 2)\) and \((-5, 5)\).

   \[
   m = \frac{y_2 - y_1}{x_2 - x_1} \\
   m = \frac{5 - 2}{-5 - (-9)} \\
   m = \frac{3}{4} \\
   
   \frac{m}{2} = \frac{3}{4}
   \]

4. Find the equation of the line with slope \( m = -3 \) and passes through \((5, -2)\).

   \[
   y - y_1 = m(x - x_1) \\
   y - (-2) = -3(x - 5) \\
   y + 2 = -3x + 15 \\
   y = -3x + 13 \\
   
   \frac{m}{2} = -3x + 13
   \]
5. Find the equation of the line with \( m = \frac{3}{4} \) and passing through \((-1, 2)\).

\[
y - y_1 = m(x - x_1)
\]
\[
y - 2 = \frac{3}{4}(x - (-1))
\]
\[
y - 2 = \frac{3}{4}x + \frac{3}{4}
\]
\[
y = \frac{3}{4}x + \frac{11}{4}
\]

7. Find the equation of the line passing through \((-7, 2)\) and has a \(y\)-intercept at 3.

NOTE: First, we must find the slope of the line. Remember that a \(y\)-intercept at 3 translates to the ordered pair \((0, 3)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{-7 - 0} = \frac{-1}{-7} = \frac{1}{7}
\]
\[
y = mx + b
\]
\[
y = \frac{1}{7}x + 3
\]

8. Find the equation of the line which has an \(x\)-intercept at \(-2\) and a \(y\)-intercept at 4.

NOTE: This means that the line passes through \((-2, 0)\) and \((0, 4)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2
\]
\[
y = mx + b
\]
\[
y = 2x + 4
\]

9. Find the equation of the line passing through \((-7, 2)\) and has an \(x\)-intercept at 3.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-7 - 3} = \frac{2}{-10} = -\frac{1}{5}
\]
\[
y - 0 = -\frac{1}{5}(x - 3)
\]
\[
y = -\frac{1}{5}x + \frac{3}{5}
\]