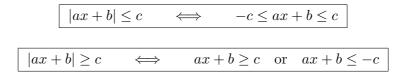
Definitions:

• Absolute value equations: Let c > 0. Then

 $|ax+b| = c \quad \iff \quad ax+b = c \quad \text{or} \quad ax+b = -c$

• Absolute value inequalities: For c > 0,



Important Properties:

• Addition Property of Inequality: If a, b, and c are real numbers, then

a < b and a + c < b + c

are equivalent. (That is, you can add or subtract the same quantity on both sides of the inequality without changing the solution.)

- Multiplication Property of Inequality: For all real numbers a, b, and c, with $c \neq 0$,
 - 1. a < b and ac < bc are equivalent if c > 0.
 - 2. a < b and ac > bc are equivalent if c < 0.

(That is, whenever you multiply or divide by a negative number you must reverse or flip the inequality.)

- To solve an absolute value equation or inequality rewrite it without the absolute value using the definitions given above.
- To solve |ax + b| = |cx + d|, rewrite as

ax + b = cx + d or ax + b = -(cx + d).

Solve these two equations for the answers.

Common Mistakes to Avoid:

- Before rewriting the absolute value equation or inequality, make sure the absolute value is isolated on one side. Do NOT rewrite until the absolute value is isolated.
- When solving $|ax + b| \le c$, the answer must be written as a three-part inequality. Do NOT break up the answer into two inequalities.
- When solving $|ax + b| \ge c$, the answer must be written as two inequalities. Do NOT combine into one three-part inequality.

PROBLEMS

Solve for x in each of the following equations or inequalities.

1. |x-3| = 4

Since the absolute value is already isolated, we will rewrite the equation.

T

$$\begin{array}{c|c} x-3 = 4 \\ x = 7 \end{array} & x-3 = -4 \\ x = -1 \end{array}$$

$$x = 7, \quad x = -1$$

2. |2 - 3x| - 5 = 7

First, we need to isolate the absolute value.

$$|2 - 3x| - 5 = 7$$
$$|2 - 3x| = 12$$

Rewriting the expression, we get

$$2 - 3x = 12$$

$$-3x = 10$$

$$x = -\frac{10}{3}$$

$$2 - 3x = -12$$

$$-3x = -14$$

$$x = \frac{14}{3}$$

$$x = -\frac{10}{3}, \quad x = \frac{14}{3}$$

3. |2x+3| = 5

Since the absolute value is already isolated, we will rewrite the equation.

$$2x + 3 = 5$$

$$2x = 2$$

$$x = 1$$

$$2x + 3 = -5$$

$$2x = -8$$

$$x = -4$$

$$x = -4$$

4.
$$5 - |4x + 1| = 2$$

First, we need to isolate the absolute value.

$$5 - |4x + 1| = 2$$
$$-|4x + 1| = -3$$
$$|4x + 1| = 3$$

Rewriting the expression, we get

$$\begin{array}{c}
4x + 1 = 3 \\
4x = 2 \\
x = \frac{2}{4} \\
x = \frac{1}{2}
\end{array}$$

$$\begin{array}{c}
4x + 1 = -3 \\
4x = -4 \\
x = -1 \\
x = -1
\end{array}$$

$$x = \frac{1}{2}, \quad x = -1$$

Absolute value equations and inequalities, page 3

5. |3x - 7| = |4x + 2|

Rewriting the equation, we get

$$3x - 7 = 4x + 2$$

$$-x - 7 = 2$$

$$-x = 9$$

$$x = -9$$

$$x = -9$$

$$3x - 7 = -(4x + 2)$$

$$3x - 7 = -4x - 2$$

$$7x - 7 = -2$$

$$7x = 5$$

$$x = \frac{5}{7}$$

6. |2x - 3| < 5

Rewriting this inequality, we get

$$-5 < 2x - 3 < 5$$

-2 < 2x < 8
-1 < x < 4
$$-1 < x < 4$$

7. $|3x+5| \ge 7$

Rewriting this expression, we get

$$3x + 5 \ge 7$$

$$3x \ge 2$$

$$x \ge \frac{2}{3}$$

$$3x + 5 \le -7$$

$$3x \le -12$$

$$x \le -4$$

$$x \ge \frac{2}{3}, x \le -4$$

8. $|2 - 5x| - 3 \le 9$

First, we need to isolate the absolute value.

$$\begin{aligned} |2-5x|-3 &\leq 9 \\ |2-5x| &\leq 12 \end{aligned}$$

Rewriting this inequality, we get

$$-12 \le 2 - 5x \le 12$$
$$-14 \le -5x \le 10$$
$$\frac{14}{5} \ge x \ge -2$$
$$-2 \le x \le \frac{14}{5}$$

9. 5 - |2x + 4| < 1

First, we need to isolate the absolute value.

$$5 - |2x + 4| < 1$$

-|2x + 4| < -4
 $|2x + 4| > 4$

Rewriting the inequality, we get

$$2x + 4 > 4
2x > 0
x > 0
2x + 4 < -4
2x < -8
x < -4$$

$$x > 0, \ x < -4$$