Definitions:

- **Rational Expression:** is the quotient of two polynomials. For example,
  \[ \frac{x}{y}, \quad \frac{x + 1}{3x - 2}, \quad \frac{x^2 - 3x + 4}{x^6 - 3} \]
  are all rational expressions.

- **Lowest terms:** A rational expression is in lowest terms when the numerator and denominator contain no common factors.

Important Properties:

- **To add or subtract rational expressions:** you MUST have a common denominator. Therefore, factor each denominator first to find a common denominator. Then you can add (or subtract) the terms and simplify.

- Remember that \( \frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \).

- To find the common denominator, it is NOT always necessary to multiply all denominators together.

- Remember that \( a - b = -(b - a) \).

Common Mistakes to Avoid:

- To add and subtract rational expressions you MUST have a common denominator. Be aware that
  \[ \frac{1}{a} + \frac{1}{b} \neq \frac{1}{a+b}. \]

- When subtracting rational expressions remember to distribute the subtraction sign to every term in the numerator of the fraction that follows it. For example,
  \[ \frac{x}{x-2} - \frac{x + 1}{x-2} = \frac{x - (x + 1)}{x-2} = \frac{x - x - 1}{x-2} = -\frac{1}{x-2}. \]

- \( \frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c} \).

- You CANNOT obtain a common denominator by adding (or subtracting) the same constant in the numerator and denominator. Therefore,
  \[ \frac{a}{b} \neq \frac{a+c}{b+c}. \]
PROBLEMS

Perform the indicated operations and simplify.

1. \[
\frac{2x + 3}{x + 1} + \frac{3x + 2}{x + 1} = \frac{2x + 3 + 3x + 2}{x + 1} = \frac{5x + 5}{x + 1} = 5
\]

2. \[
\frac{3}{10x + 15} - \frac{5}{12x + 18}
\]

\[
\frac{3}{10x + 15} - \frac{5}{12x + 18} = \frac{3(12x + 18) - 5(10x + 15)}{12x + 18(10x + 15)} = \frac{36x + 54 - 50x - 75}{120x^2 + 105x} = \frac{-14x - 21}{120x^2 + 105x}
\]

3. \[
\frac{8}{x - 4} + \frac{2}{4 - x} = \frac{8}{x - 4} + \frac{2}{-(x - 4)} = \frac{8}{x - 4} - \frac{2}{x - 4} = \frac{6}{x - 4}
\]

4. \[
\frac{2x}{x + 4} + \frac{3}{x - 7}
\]

\[
\frac{2x}{x + 4} + \frac{3}{x - 7} = \frac{2x(x - 7) + 3(x + 4)}{(x + 4)(x - 7)} = \frac{2x^2 - 14x + 3x + 12}{(x + 4)(x - 7)} = \frac{2x^2 - 11x + 12}{(x + 4)(x - 7)} = \frac{(2x - 3)(x - 4)}{(x + 4)(x - 7)}
\]
5. \[ \frac{2}{x+3} - \frac{1}{x^2+7x+12} \]

\[ \frac{2}{x+3} - \frac{1}{(x+4)(x+3)} \]

\[ \frac{2(x+4)}{(x+3)(x+4)} - \frac{1}{(x+4)(x+3)} \]

\[ \frac{2x+8 - 1}{(x+3)(x+4)} \]

\[ \frac{2x+7}{(x+3)(x+4)} \]

6. \[ \frac{x}{(x+2)^2} + \frac{3}{x+2} \]

\[ \frac{x}{(x+2)(x+2)} + \frac{3(x+2)}{(x+2)(x+2)} \]

\[ \frac{x}{(x+2)(x+2)} + \frac{3x+6}{(x+2)(x+2)} \]

\[ \frac{x+3x+6}{(x+2)(x+2)} \]

\[ \frac{4x+6}{(x+2)(x+2)} \]

\[ \frac{2(2x+3)}{(x+2)(x+2)} \]

7. \[ \frac{2}{x-5} - \frac{1}{x} - \frac{5}{x^2-5x} \]

\[ \frac{2}{x-5} - \frac{1}{x} - \frac{5}{x(x-5)} \]

\[ \frac{2x}{x(x-5)} - \frac{(x-5)}{x(x-5)} - \frac{5}{x(x-5)} \]

\[ \frac{2x - (x-5) - 5}{x(x-5)} \]

\[ \frac{2x - x + 5 - 5}{x(x-5)} \]

\[ \frac{x}{x(x-5)} \]

\[ \frac{1}{x-5} \]
8. \( \frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} \)

\[
\frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} = \frac{x(x - 4)}{(x + 2)(x - 1)(x - 4)} - \frac{2(x + 2)}{(x + 2)(x - 1)(x - 4)}
\]

\[
= \frac{x^2 - 4x - (2x + 4)}{(x + 2)(x - 1)(x - 4)}
\]

\[
= \frac{x^2 - 4x - 2x - 4}{(x + 2)(x - 1)(x - 4)}
\]

\[
= \frac{x^2 - 6x - 4}{(x + 2)(x - 1)(x - 4)}
\]

9. \( \frac{4x}{x - 1} - \frac{2}{x + 1} - \frac{4}{x^2 - 1} \)

\[
\frac{4x}{x - 1} - \frac{2}{x + 1} - \frac{4}{x^2 - 1} = \frac{4x(x + 1)}{(x - 1)(x + 1)} - \frac{2(x - 1)}{(x - 1)(x + 1)} - \frac{4}{(x - 1)(x + 1)}
\]

\[
= \frac{4x^2 + 4x - (2x - 2) - 4}{(x - 1)(x + 1)}
\]

\[
= \frac{4x^2 + 4x - 2x + 2 - 4}{(x - 1)(x + 1)}
\]

\[
= \frac{4x^2 + 2x - 2}{(x - 1)(x + 1)}
\]

\[
= \frac{2(2x^2 + x - 1)}{(x - 1)(x + 1)}
\]

\[
= \frac{2(2x - 1)(x + 1)}{(x - 1)(x + 1)}
\]

\[
= \frac{2(2x - 1)}{x - 1}
\]
10. \[ \frac{2x}{x+3} - \frac{8}{x^2 + 8x + 15} - \frac{4}{x+5} \]

\[ \frac{2x}{x+3} - \frac{8}{x^2 + 8x + 15} - \frac{4}{x+5} \]

\[ \frac{2x}{x+3} - \frac{8}{(x+5)(x+3)} - \frac{4}{x+5} \]

\[ \frac{2x(x+5)}{(x+5)(x+3)} - \frac{8}{(x+5)(x+3)} - \frac{4(x+3)}{(x+5)(x+3)} \]

\[ \frac{2x^2 + 10x}{(x+5)(x+3)} - \frac{8}{(x+5)(x+3)} - \frac{4(x+12)}{(x+5)(x+3)} \]

\[ \frac{2x^2 + 10x - 8 - (4x + 12)}{(x+5)(x+3)} \]

\[ \frac{2x^2 + 10x - 8 - 4x - 12}{(x+5)(x+3)} \]

\[ \frac{2x^2 + 6x - 20}{(x+5)(x+3)} \]

\[ \frac{2(x^2 + 3x - 10)}{(x+5)(x+3)} \]

\[ \frac{2(x+5)(x-2)}{(x+5)(x+3)} \]

\[ \frac{2(x-2)}{x+3} \]