

**Definitions:**

- **System of linear equations:** consists of two or more linear equations with the same variables.
- **Consistent:** The system is consistent if there is exactly one solution.
- **Inconsistent:** The system is inconsistent if there is no solution. This happens when the two equations represent parallel lines.
- **Dependent:** The system is dependent if there is an infinite number of ordered pairs as solutions. This occurs when the two equations represent the same line.

**Steps for the Method of Elimination:**

1. Align the variables.
2. Multiply one or both equations by the appropriate constant so that the coefficients of one of the variables are opposites of each other.
3. Add the two equations together. (If you have done everything correctly, this will produce one equation with one variable).
4. Solve the equation from Step 3.
5. Substitute the solution from Step 4 into either of the original equations. This will give the value of the other variable.

**Important Properties:**

- If the equation in Step 3 above is a false statement (such as  $7 = 2$ ), then the system is inconsistent.
- If the equation in Step 3 above is a true statement (such as  $0 = 0$ ), then the system is dependent.

**Common Mistakes to Avoid:**

- Remember that a system of linear equations is not completely solved until values for both  $x$  and  $y$  are found. To avoid this mistake, write all answers as ordered pairs.
- Remember that all ordered pairs are stated with the  $x$ -variable first and the  $y$ -variable second; namely,  $(x, y)$ .

## PROBLEMS

1. Solve

$$6x - 5y = 25$$

$$4x + 15y = 13$$

If we multiply the first equation by 3 and leave the second equation alone, we will eliminate  $y$ .

$$3(6x - 5y = 25)$$

$$4x + 15y = 13$$

We now have:

$$18x - 15y = 75$$

$$4x + 15y = 13$$

Adding these two equations together we get:

$$22x = 88$$

$$x = 4$$

Now, we must find the value for  $y$  by substituting  $x = 4$  into one of the two original equations. Substituting into the first equation gives

$$6(4) - 5y = 25$$

$$24 - 5y = 25$$

$$-5y = 1$$

$$y = -\frac{1}{5}$$

Answer: $\left(4, -\frac{1}{5}\right)$
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2. Solve

$$5x - 2y = 16$$

$$2x + y = 8$$

Multiplying the second equation by 2 will eliminate the  $y$  variable.

$$5x - 2y = -16$$

$$2(2x + y = 8)$$

This gives us:

$$5x - 2y = -16$$

$$4x + 2y = 16$$

When we add these two equations together we get:

$$9x = 0$$

$$x = 0$$

Now, we must find the value of  $y$  by substituting  $x = 0$  into one of the two original equations. When we substitute  $x = 0$  into the second equation, we get

$$2(0) + y = 8$$

$$y = 8$$

Answer: $(0, 8)$
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3. Solve

$$\begin{aligned}8x + 9y &= 13 \\6x - 5y &= 45\end{aligned}$$

To eliminate the  $x$  variable we will multiply the first equation by  $-3$  and the second equation by  $4$ .

$$\begin{aligned}-3(8x + 9y &= 13) \\4(6x - 5y &= 45)\end{aligned}$$

This gives us:

$$\begin{aligned}-24x - 27y &= -39 \\24x - 20y &= 180\end{aligned}$$

Adding these two equations together yields:

$$\begin{aligned}-47y &= 141 \\y &= -3\end{aligned}$$

Next, we need to find the value for  $x$  by substituting  $y = -3$  into either of the two original equations. If we substitute  $y = -3$  into the first equation, we get

$$\begin{aligned}8x + 9(-3) &= 13 \\8x - 27 &= 13 \\8x &= 40 \\x &= 5\end{aligned}$$

Answer: $(5, -3)$
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4. Solve

$$\begin{aligned}4x - 5y &= 35 \\3x - 4y &= 24\end{aligned}$$

If we multiply the first equation by  $3$  and the second equation by  $-4$  we will eliminate the  $x$  variable.

$$\begin{aligned}3(4x - 5y &= 35) \\-4(3x - 4y &= 24)\end{aligned}$$

This yields:

$$\begin{aligned}12x - 15y &= 105 \\-12x + 16y &= -96\end{aligned}$$

When we add these two equations together, we get:

$$y = 9$$

Next, to solve for the  $x$  variable we need to substitute  $y = 9$  into one of the two original equations. Substituting  $y = 9$  into the second equation, we get:

$$\begin{aligned}3x - 4(9) &= 24 \\3x - 36 &= 24 \\3x &= 60 \\x &= 20\end{aligned}$$

Answer: $(20, 9)$
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5. Solve

$$\begin{aligned} -6x + 9y &= 12 \\ 2x - 3y &= -4 \end{aligned}$$

If we multiply the second equation by 3 we will eliminate the  $x$  variable.

$$\begin{aligned} -6x + 9y &= 12 \\ 3(2x - 3y) &= -4 \end{aligned}$$

This gives us:

$$\begin{aligned} -6x + 9y &= 12 \\ 6x - 9y &= -12 \end{aligned}$$

When we add these two equations together, we get:

$$0 = 0$$

Since this is a true statement, we know that the system is dependent. Therefore, there are an infinite number of solutions.

Answer: dependent system
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6. Solve

$$\begin{aligned} -5x + 4y &= 1 \\ 15x - 12y &= 4 \end{aligned}$$

If we multiply the first equation by 3 we will eliminate the  $x$  variable.

$$\begin{aligned} 3(-5x + 4y) &= 3(1) \\ 15x - 12y &= 4 \end{aligned}$$

This gives us:

$$\begin{aligned} -15x + 12y &= 3 \\ 15x - 12y &= 4 \end{aligned}$$

Adding these two equations together gives us:

$$0 = 7$$

Since this is a false statement, we know that the system is inconsistent. Therefore, there is no solution.

Answer: No Solution
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