A function \( f \) is a rule that assigns to every element \( x \) contained in a set \( A \), \textit{exactly one} element \( y \) in a set \( B \). Another way to think of a function is the phrase “for every \( x \), there is only one \( y \).”

The symbol \( f(x) \), read “\( f \) of \( x \),” is called the \textbf{value of the function at} \( x \) and is usually equated with the variable \( y \). In other words, we write \( y = f(x) \).

The set \( A \) is called the \textbf{domain} of the function. The domain of a function is the set of all values of \( x \) for which the function is defined. If \( x \) is any element in the domain, then \( x \) is called the \textbf{independent variable}. The domain can also be thought of as the set of all input values.

The \textbf{range} of the function is the set of all possible values of \( f(x) \), as \( x \) varies throughout the domain. Hence, the range of a function is the set of all \( y \) values assumed by the function. If \( y \) represents an output of the function \( f \) from an input \( x \), then \( y \) is called the \textbf{dependent variable}.

A function may be defined by a set of ordered pairs, a table, an arrow diagram, a graph, or an equation. Because functions play an important role in mathematics, it is important to recognize when a particular relationship represents a function.

**Example 1:** Determine which of the following are examples of functions. For each function, determine the domain and range.

(a) \( \{(1, 2), (3, 6), (6, 8), (9, 2), (12, 5)\} \)

(b) \[
\begin{array}{c|cccc}
  x & 1 & 2 & -5 & 2 \\
  y & -1 & 4 & 6 & 7 \\
\end{array}
\]

(c) \[
\begin{array}{c}
1 \\
5 \\
2 \\
3 \\
3 \\
\end{array} \rightarrow \begin{array}{c}
1 \\
4 \\
9 \\
\end{array}
\]

(d) \[
\begin{array}{c}
1 \\
2 \\
3 \\
\end{array} \rightarrow \begin{array}{c}
1 \\
4 \\
9 \\
\end{array}
\]

(e) \( x + y^2 = 9 \) where \( x \) is the input.
• The **graph of a function** is a set of points \((x, y)\) in the \(xy\)-plane such that \(y = f(x)\).

• **The Vertical Line Test:** A set of points in the \(xy\)-plane is the graph of a function if and only if no vertical line intersects the set of points more than once.

**Example 2:** Determine if each of following curves is the graph of a function.

![Graphs](image)

• In our definition of a function \(y = f(x)\), the independent variable \(x\) serves as a placeholder for all input values. Therefore, to evaluate a function at a number, we substitute the number for the placeholder.

**Example 3:** Consider the function \(f(x) = 3x^2 - 2x - 8\). Find

(a) \(f(0)\)

(b) \(f(-1)\)

(c) \(f(2)\)

(d) \(f\left(\frac{1}{2}\right)\)