MATH 10771: Exam #3 (Fall 2016)

1. Use divisibility tests to determine if

   8734265436

   is divisible by

   2, 3, 4, 5, 6, 8, 9, 10, 12

   DO NOT USE A CALCULATOR. **State why or why not for each number**.

2. Simplify each expression. Exact answers only.

   (a) $(-4)^4 =$ (b) $(-3)^{-4} =$ (c) $-7^2 =$ (d) $-8^{-3} =$

3. Devise a divisibility test for 24. Be specific and state the divisibility test completely.

4. Find all integers n, $-30 \leq n \leq 30$, which make the following congruences true.

   (a) $19 \equiv -29 \pmod{n}$ (b) $15 \equiv n \pmod{6}$

5. Determine the remainder when 3^{127} is divided by 5. You must show all work. Answers without work will receive no credit.

6. Illustrate $(-3) \times (-5)$ using the charge field method. Explain your example.

7. If $b = 2^2 \cdot 3^5 \cdot 5 \cdot 7^3 \cdot 11 \cdot 13^2 \cdot 17$ and $\text{GCF}(a, b) = 2^2 \cdot 3^2 \cdot 5 \cdot 11 \cdot 13^2$ and $\text{LCM}(a, b) = 2^3 \cdot 3^5 \cdot 5^3 \cdot 7^3 \cdot 11^2 \cdot 13^4 \cdot 17$ find a.

8. How many factors does $n = 2^5 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11^2 \cdot 13^4$ have?

9. If a is negative, b is negative, and c is positive, determine whether each of the following is positive, negative, or cannot be determined.

   (a) $ac - (a + b)$ (c) $(2a + 3b)(4c) - b^2$

   (b) $-a(b - c)(a - c)$ (d) $bc - a^2 - ac$

10. CHOOSE ONE!!! Use divisibility tests to determine whether

    $$36074973543946821$$

    is divisible by 7 or divisible by 11. State why or why not. Show all work.

11. Calculate the following in the indicated clock.

    (a) $8 \otimes 6$ (11 clock)

    (b) $7 \ominus 26$ (30 clock)

    (c) $3 \div 5$ (7 clock)

    (d) $13 \oplus 18$ (20 clock)

12. find the **LCM**(72, 120, 378).

13. Find the prime factorization for 79380.

14. Find the **LCM**(67914, 79380). (Note: 79380 is from the previous problem.)

15. Find the **GCF**(1421, 1827, 2523).

16. Short answer.

    (a) Illustrate $3 + (-6)$ using the measurement model.

    (b) In a 20-clock, what is the additive identity?

    (c) In an 9-clock, find the reciprocal of 2.

    (d) Name a property that integer subtraction has that whole number subtraction did not.

    (e) A counting number with more than two factors is called a(n) _____.

    (f) In a 26-clock, find the additive inverse of 17.

    (g) To determine if 467 is prime, we must check to see if it is divisible by any numbers other than 1 and itself. According to the Prime Factor Test, list all of the numbers that must be checked as possible factors to see if 817 is prime.

    (h) Name a property that integer addition has that whole number addition did not.

ANSWERS

1. Let $n = 8734265436$
   $2|n$ since n ends in even number
   $3|n$ since $3|48$ (48 is sum of digits)
   $4|n$ since $4|36$
   5 does not divide n since n does not end in 0 or 5
   $6|n$ since $2|n$ and $3|n$
   8 does not divide n since 8 does not divide 436
   9 does not divide n since 9 does not divide 48 (the sum of the digits)
   10 does not divide n since n does not end in 0
   $12|n$ since $3|n$ and $4|n$

2. (a) 256 (b) $\frac{1}{81}$ (c) -49 (d) $-\frac{1}{512}$

3. $24|n$ if and only if $3|n$ and $8|n$.

4. (a) $n \in \{2, 3, 4, 6, 8, 12, 16, 24\}$ (b) $\{-27, -21, -15, -9, -3, 3, 9, 15, 21, 27\}$

5. 2

6. Illustrate by removing 3 groups of five negatives.

7. $a = 2^3 \cdot 3^2 \cdot 5^3 \cdot 11^2 \cdot 13^4$

8. 1080 factors

9. (a) cannot be determined (b) positive (c) negative (d) cannot be determined

10. yes, the number is divisible by 11.

11. (a) 4 (b) 11 (c) 2 (d) 11

12. 7560

13. $2^2 \cdot 3^4 \cdot 5 \cdot 7^2$

14. $2^2 \cdot 3^4 \cdot 5 \cdot 7^3 \cdot 11$

15. 29

16. (a) on a number line move 3 spaces to the right and then move six spaces to the left.
    (b) 20 or 0 (c) 5 (d) closure property
    (e) composite
    (f) 2, 3, 5, 7, 11, 13, 17, 19
    (g) additive inverse property