
Math 10771: Radicals

Example 1. Simplify: $\sqrt{162}$

SOLUTION.

$$\begin{aligned}\sqrt{162} &= \sqrt{81} \cdot \sqrt{2} \\ &= \boxed{9\sqrt{2}}\end{aligned}$$

Example 2. Add/Subtract: $\sqrt{12} - 3\sqrt{8} + 4\sqrt{27}$

SOLUTION.

$$\begin{aligned}\sqrt{12} - 3\sqrt{8} + 4\sqrt{27} &= \sqrt{4 \cdot 3} - 3\sqrt{4 \cdot 2} + 4\sqrt{9 \cdot 3} \\ &= 2\sqrt{3} - 3(2)\sqrt{2} + 4(3)\sqrt{3} \\ &= 2\sqrt{3} - 6\sqrt{2} + 12\sqrt{3} \\ &= \boxed{14\sqrt{3} - 6\sqrt{2}}\end{aligned}$$

Example 3. Multiply: $(5\sqrt{2} + \sqrt{3})(3\sqrt{2} + \sqrt{3})$

SOLUTION. Use FOIL and note that $\sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2$:

$$\begin{aligned}(5\sqrt{2} + \sqrt{3})(3\sqrt{2} + \sqrt{3}) &= 15\sqrt{4} + 5\sqrt{6} + 3\sqrt{6} + \sqrt{9} \\ &= 30 + 8\sqrt{6} + 3 \\ &= \boxed{33 + 8\sqrt{6}}\end{aligned}$$

Example 4. Multiply: $(\sqrt{2} - 7)^2$

SOLUTION. You must first write the binomial twice, then use FOIL:

$$\begin{aligned}(\sqrt{2} - 7)^2 &= (\sqrt{2} - 7)(\sqrt{2} - 7) \\ &= 2 - 7\sqrt{2} - 7\sqrt{2} + 49 \\ &= \boxed{51 - 14\sqrt{2}}\end{aligned}$$

Example 5. Simplify—rationalize the denominator where necessary: $\sqrt{\frac{5}{11}}$

SOLUTION.

$$\begin{aligned}\sqrt{\frac{5}{11}} &= \frac{\sqrt{5}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} \\ &= \boxed{\frac{\sqrt{55}}{11}}\end{aligned}$$

EXERCISES

Simplify:

1. $\sqrt{24}$

7. $3\sqrt{12}$

13. $\sqrt[3]{16}$

2. $\sqrt{18}$

8. $4\sqrt{18}$

14. $\sqrt[3]{54}$

3. $\sqrt{32}$

9. $2\sqrt{8}$

15. $2\sqrt[4]{32}$

4. $\sqrt{40}$

10. $3\sqrt{28}$

16. $5\sqrt[4]{243}$

5. $2\sqrt{16}$

11. $8\sqrt{363}$

17. $\sqrt[3]{40}$

6. $4\sqrt{20}$

12. $2\sqrt{48}$

Add/subtract and simplify:

18. $5\sqrt{4} - 3\sqrt{9}$

26. $3\sqrt{3} + \sqrt{27} - 8\sqrt{75}$

19. $3\sqrt{3} - 5\sqrt{27}$

27. $\sqrt{3} - 2\sqrt{12} + 6\sqrt{3}$

20. $-2\sqrt{8} + 5\sqrt{32}$

28. $\sqrt{18} + \sqrt{8} - \sqrt{32}$

21. $5\sqrt{18} - 2\sqrt{75}$

29. $\sqrt{48} + \sqrt{20} - \sqrt{27} + 2\sqrt{20}$

22. $3\sqrt{12} - 5\sqrt{27}$

30. $\sqrt{18} + \sqrt{32} + \sqrt{50}$

23. $8\sqrt{8} - 4\sqrt{32} - 9\sqrt{50}$

24. $-2\sqrt{3} + 5\sqrt{27} - 4\sqrt{45}$

31. $\sqrt{25} + \sqrt{12}$

25. $\sqrt{25} - \sqrt{9} + \sqrt{16}$

32. $\sqrt{49} - \sqrt{90}$

Multiply:

33. $\sqrt{3}\sqrt{6}$

42. $(\sqrt{3} + 4)(\sqrt{3} - 4)$

34. $\sqrt{5}\sqrt{10}$

43. $(\sqrt{3} + 5)^2$

35. $\sqrt{2}(\sqrt{2} - \sqrt{3})$

44. $(2\sqrt{2} - 3)^2$

36. $3(\sqrt{12} - \sqrt{3})$

45. $(3 + \sqrt{2})(2 - \sqrt{3})$

37. $\sqrt{8}(\sqrt{2} - \sqrt{5})$

46. $(\sqrt{5} - 4)(\sqrt{5} + 4)$

38. $\sqrt{3}(\sqrt{5} - 2)$

47. $(\sqrt{6} + 3)^2$

39. $\sqrt{5}(\sqrt{10} - \sqrt{2})$

40. $(3\sqrt{5} - 2)(5\sqrt{5} - 4)$

48. $(\sqrt{2} + 4)(\sqrt{2} - 1)$

41. $(5\sqrt{3} + 2\sqrt{2})(3\sqrt{3} - \sqrt{2})$

49. $(4\sqrt{5} + \sqrt{3})^2$

Simplify—rationalize the denominator where necessary:

50. $\frac{8}{\sqrt{6}}$

61. $\frac{3}{\sqrt{12}}$

51. $\frac{3}{\sqrt{3}}$

56. $\sqrt{\frac{2}{3}}$

62. $\sqrt{\frac{5}{9}}$

52. $\frac{1}{\sqrt{8}}$

57. $\sqrt{\frac{3}{5}}$

63. $\sqrt{\frac{8}{16}}$

53. $\frac{6}{\sqrt{12}}$

58. $\frac{6}{\sqrt{18}}$

64. $\sqrt{\frac{3}{8}}$

54. $\sqrt{\frac{11}{4}}$

59. $\frac{4}{\sqrt{6}}$

65. $\sqrt{\frac{2}{7}}$

55. $\sqrt{\frac{8}{9}}$

60. $\frac{18}{\sqrt{12}}$

ANSWERS

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|--------------------|-------------------------------|--|
| 1. $2\sqrt{6}$ | 16. $15\sqrt[4]{3}$ | 31. $5 + 2\sqrt{3}$ |
| 2. $3\sqrt{2}$ | 17. $2\sqrt[3]{5}$ | 32. $7 - 3\sqrt{10}$ |
| 3. $4\sqrt{2}$ | 18. 1 | 33. $3\sqrt{2}$ |
| 4. $2\sqrt{10}$ | 19. $-12\sqrt{3}$ | 34. $5\sqrt{2}$ |
| 5. 8 | 20. $16\sqrt{2}$ | 35. $2 - \sqrt{6}$ |
| 6. $8\sqrt{5}$ | 21. $15\sqrt{2} - 10\sqrt{3}$ | 36. $3\sqrt{3}$ |
| 7. $6\sqrt{3}$ | 22. $-9\sqrt{3}$ | 37. $4 - 2\sqrt{10}$ |
| 8. $12\sqrt{2}$ | 23. $-45\sqrt{2}$ | 38. $\sqrt{15} - 2\sqrt{3}$ |
| 9. $4\sqrt{2}$ | 24. $13\sqrt{3} - 12\sqrt{5}$ | 39. $5\sqrt{2} - \sqrt{10}$ |
| 10. $6\sqrt{7}$ | 25. 6 | 40. $83 - 22\sqrt{5}$ |
| 11. $88\sqrt{3}$ | 26. $-34\sqrt{3}$ | 41. $41 + \sqrt{6}$ |
| 12. $8\sqrt{3}$ | 27. $3\sqrt{3}$ | 42. -13 |
| 13. $2\sqrt[3]{2}$ | 28. $\sqrt{2}$ | 43. $28 + 10\sqrt{3}$ |
| 14. $3\sqrt[3]{2}$ | 29. $\sqrt{3} + 6\sqrt{5}$ | 44. $17 - 12\sqrt{2}$ |
| 15. $2\sqrt[4]{2}$ | 30. $12\sqrt{2}$ | 45. $6 - 3\sqrt{3} + 2\sqrt{2} - \sqrt{6}$ |

46. -11

47. $15 + 6\sqrt{6}$

48. $-2 + 3\sqrt{2}$

49. $83 + 8\sqrt{15}$

50. $\frac{4\sqrt{6}}{3}$

51. $\sqrt{3}$

52. $\frac{\sqrt{2}}{4}$

53. $\sqrt{3}$

54. $\frac{\sqrt{11}}{2}$

55. $\frac{2\sqrt{2}}{3}$

56. $\frac{\sqrt{6}}{3}$

57. $\frac{\sqrt{15}}{5}$

58. $\sqrt{2}$

59. $\frac{2\sqrt{6}}{3}$

60. $3\sqrt{3}$

61. $\frac{\sqrt{3}}{2}$

62. $\frac{\sqrt{5}}{3}$

63. $\frac{\sqrt{2}}{2}$

64. $\frac{\sqrt{6}}{4}$

65. $\frac{\sqrt{14}}{7}$