Section 2.1: Sets

- **Set:** A **set** is a collection of objects. A set is usually denoted with a capital letter.

- **Element:** An **element** is an object in the set.

- **Rules regarding sets:** The same element is not listed more than once within a set and the order of the elements in the set is not important.

THREE DIFFERENT WAYS TO DEFINE A SET:

1. **Verbal Description:** The set of all states in the United States which start with the letter O.

2. **Listing method:** Listing the elements separated by commas. $\{Oklahoma, Ohio, Oregon\}$.

3. **Set-Builder Notation:** $\{x \mid x$ is a US state which begins with the letter O$\}$.

NOTATION:

- $\in$ denotes that an object is in the set.

- $\notin$ denotes that the object is NOT in the set.

- $\emptyset$ or $\{\}$ denote the **empty set**. The empty set is a set with no elements.

BE CAREFUL! The notation $\{\emptyset\}$ does NOT represent the empty set.

OTHER DEFINITIONS:

- **Finite Set:** A set is finite if it is empty or can have its elements listed where the list eventually ends.

- **Infinite Set:** A set is infinite if it goes on without end. In other words, it is not finite.

- **Equal Sets:** Two sets A and B are equal, denoted $A = B$, if and only if they have the same elements. For example, if $A = \{x|x \text{ is an positive even number less than ten}\}$ and $B = \{2, 4, 6, 8\}$, then $A = B$.

- **Equivalent Sets:** Two finite sets A and B are equivalent, denoted $A \sim B$, if and only if they have the same number of elements. For example, if $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$, then $A \sim B$. Therefore, if $A = B$, then $A \sim B$; however, if $A \sim B$, then it is not necessarily true that $A = B$.

  A **one-to-one correspondence** between two sets A and B is a pairing of elements of A with the elements of B so that each element of A corresponds to exactly one element of B, and vice versa. If there is a 1-1 correspondence between sets A and B, then $A \sim B$. Note that every infinite set can be matched or placed in a 1-1 correspondence with a proper subset of itself.

- **Subset of a set:** Set A is said the be a subset of B, denoted $A \subseteq B$, if and only if every element of A is also an element of B. Note that $A \subseteq B$ leaves open the possibility that $A = B$.

  If A has n elements, then A has 2^n subsets.

- **Proper subset:** A is a proper subset of B, denoted $A \subset B$, if every element of A is also an element of B and B has an element that is not in A. Note that if $A \subset B$, then $A \neq B$. Also, if A is a nonempty set, then $\emptyset \subset A$.

  If A has n elements, then A has $2^n - 1$ proper subsets.

- **Disjoint sets:** A and B are disjoint if they have no elements in common.

Example 1: Determine if each of the following is true or false.

(a) $\{3\} \in \{1,2,3\}$

(b) $\{4\} \subset \{2,4,6\}$

(c) $5 \subseteq \{1,3,5,7\}$

(d) $7 \in \{3,5,7,9\}$

Example 2: Let $A = \{1,3\}$. List all subsets of A.

Example 3: Let $A = \{\square, \triangle, \bigcirc\}$. List all proper subsets of A.