Section 5.2: GCF and LCM

- **Greatest Common Factor (GCF)**: of two (or more) nonzero whole numbers is the largest whole number that is a factor of both (or all) of the numbers. The GCF of a and b is denoted $\mathrm{GCF}(a, b)$.

  1. **Set Intersection Method**: List all factors for a in one set. List all factors of b in another set. Take the intersection of these two sets. The GCF is the largest number in the intersection.

  2. **Prime Factorization Method**: Express the prime factorization of each number. The GCF is the product of the <u>common</u> primes to their <u>smallest</u> exponent.

  3. **Euclidean Algorithm**: If a and b are whole numbers with $a \geq b$ and $a = bq + r$, where $r < b$, then $\mathrm{GCF}(a, b) = \mathrm{GCF}(r, b)$.

     Thus, to find the GCF of two numbers, this theorem can be applied repeatedly until a remainder of zero is obtained. The final <u>divisor</u> that leads to the zero remainder is the GCF of the two numbers.

Example 1: Find the GCF(42, 385).

Example 2: Find the GCF(840, 3432).

Theorem 1: If a and b are whole numbers, with $a \geq b$, then

$$\mathrm{GCF}(a, b) = \mathrm{GCF}(a - b, b).$$

- **Least Common Multiple (LCM)**: of two (or more) nonzero whole numbers is the smallest nonzero whole number that is a multiple of each (or all) of the numbers. The LCM of a and b is denoted LCM(a, b).

1. **Set Intersection Method**: List the first several nonzero multiples of a in one set. List the first several nonzero multiples of b in another set. Take the intersection of these sets. The LCM is the smallest number in the intersection.

2. **Prime Factorization Method**: Express the prime factorization of each number. The LCM is the product of all primes appearing in the factorizations to their highest exponent.

3. **Division by Primes (Ladder Method)**: To find the LCM we use the following steps.

Step 1: Write the given numbers in a horizontal line, separating them by commas.

Step 2: Divide them by a suitable prime number which exactly divides at least two of the given numbers.

Step 3: We put the quotient directly under the numbers in the next row. If the number is not divided exactly, we bring it down in the next row.

Step 4: We continue the process of step 2 and step 3 until all the numbers left in the last row have only the number 1 as a common divisor.

Step 5: We multiply all the prime numbers by which we have divided and the all the numbers left in the last row. This product is the least common multiple of the given numbers.

Example 3: Find the LCM(18, 24, 60).

Example 4: Find the LCM(113400, 642096).

Example 5: Find the LCM(120, 144, 160, 180).

Theorem 2: Let a and b be any two whole numbers. Then

$$\text{GCF}(a, b) \cdot \text{LCM}(a, b) = a \cdot b.$$

Example 6: If $a = 2^3 \cdot 3^2 \cdot 5^4 \cdot 7^3$, $GCF(a, b) = 2^2 \cdot 3^2 \cdot 7^3$, and $LCM(a, b) = 2^3 \cdot 3^8 \cdot 5^4 \cdot 7^4 \cdot 11$, then find b.

Theorem 3: Suppose that a counting number n is expressed as a product of distinct primes with their respective exponents; say,

$$n = p_1^{n_1} \cdot p_2^{n_2} \cdots p_m^{n_m}$$

Then the number of factors of n is the product

$$(n_1 + 1)(n_2 + 1) \cdots (n_m + 1).$$

Example 7: How many factors does 173250 have?